Games and Logic for Cryptographic Protocol Verification

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Supervisors:

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21-01-2026

1. Introduction

Games and Logic for Cryptographic Protocol Verification

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Cryptographic protocol:
Distributed program securing communications

- Messaging (e.g. Signal): Confidentiality
- Internet navigation (e.g. TLS): Authentication
- E-voting: Privacy

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Verification:
Several layers of verifications:
High-level specification

Games and logic

- Formal methods
- Cryptographic reductions
- Computational model ($\neq$ symbolic model)

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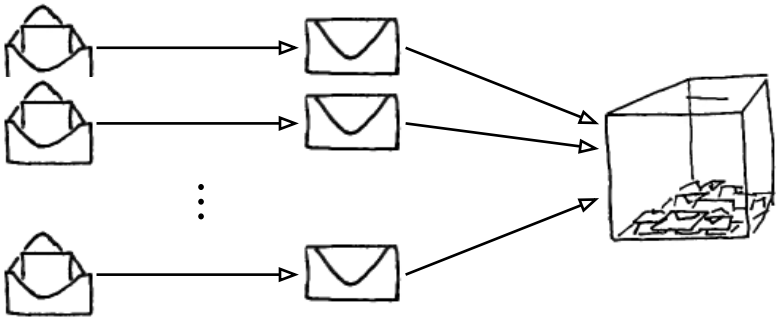
Several layers of verifications:
High-level specification

2. This thesis

Running example: voting

Collect phase:



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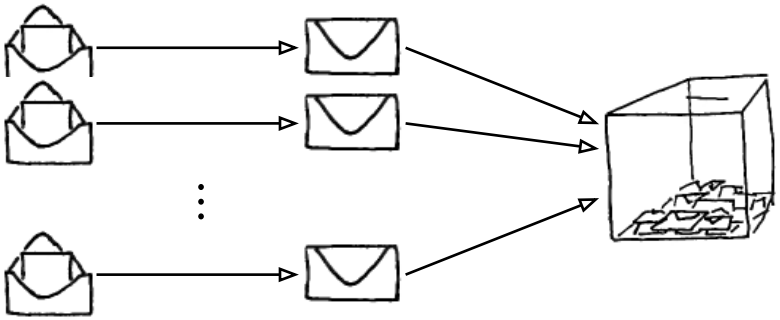
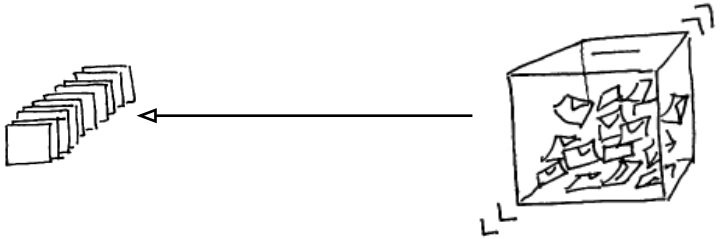
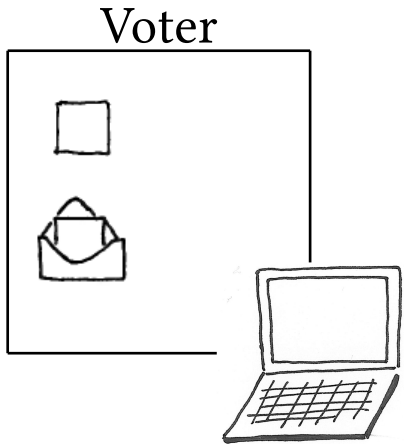


Publish phase:



Running example: very simplified voting protocol



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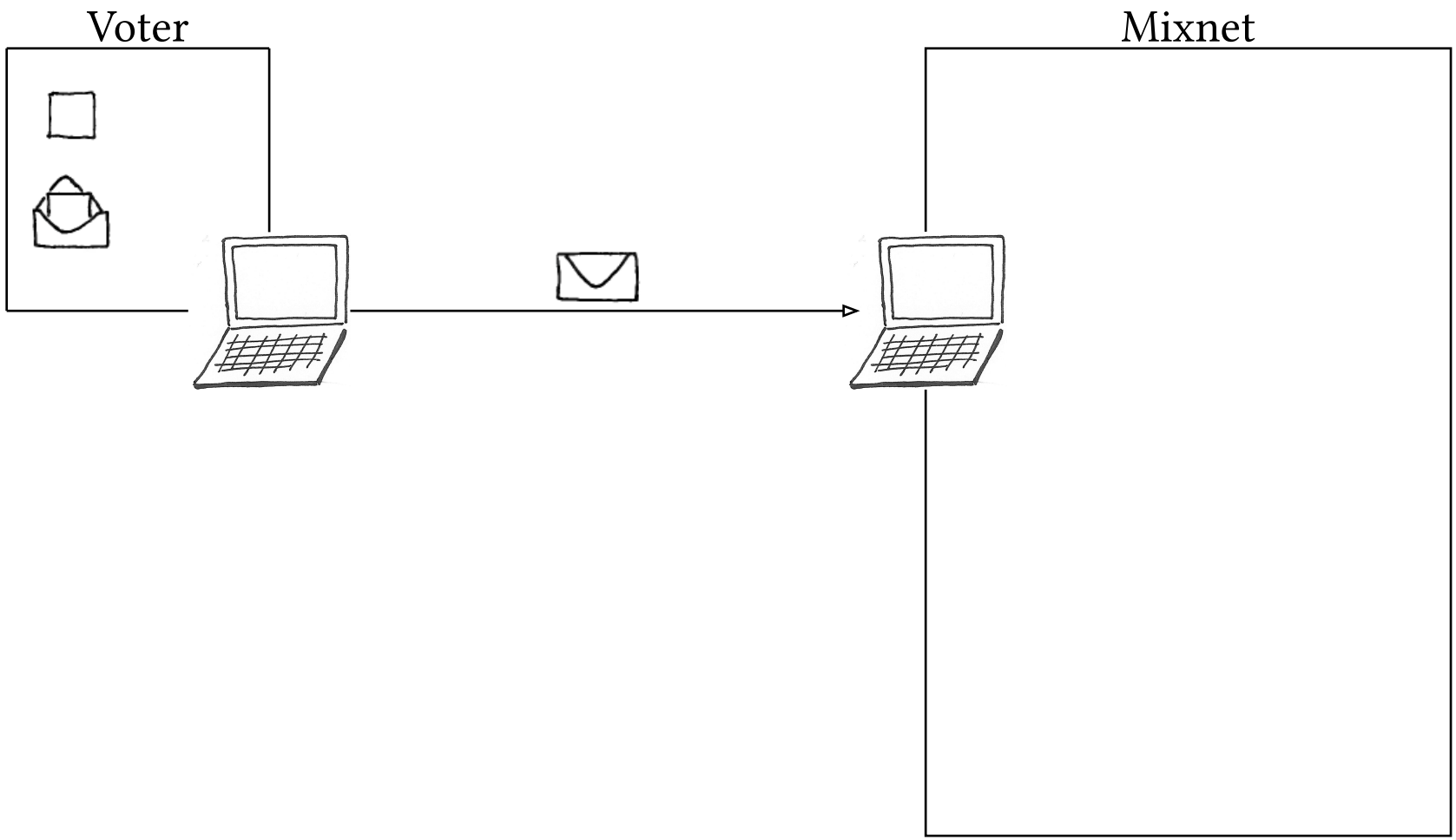

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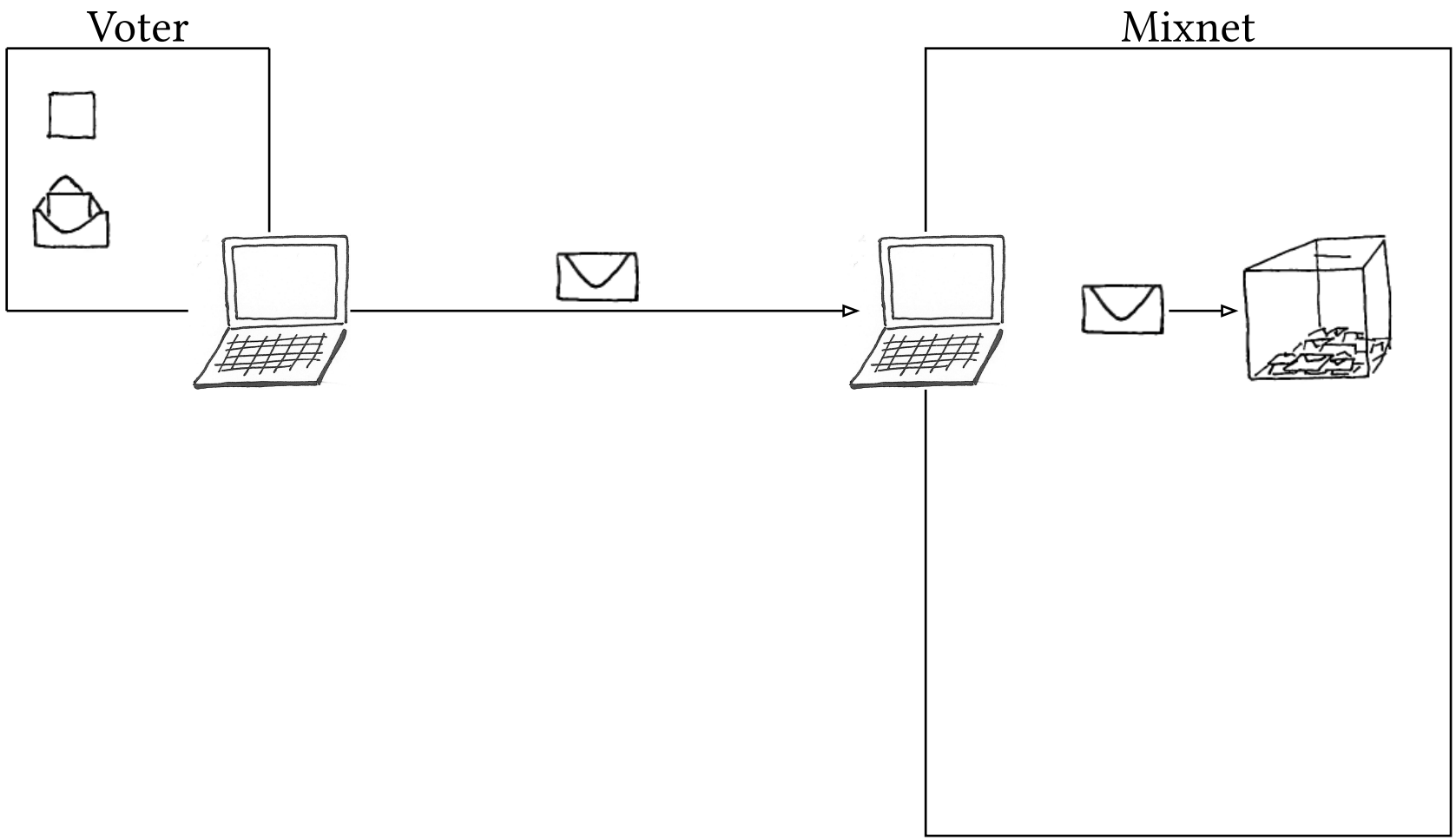

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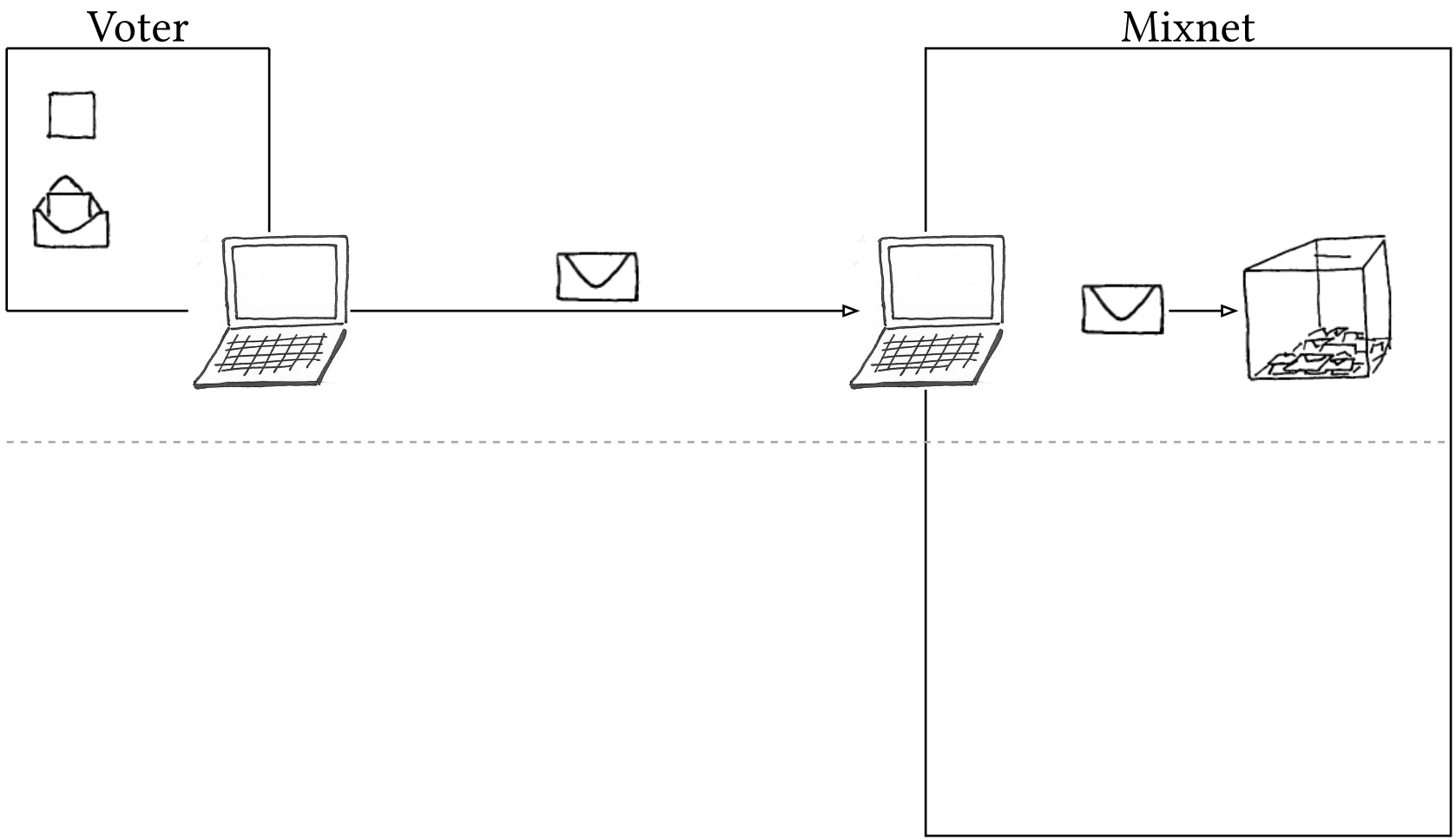

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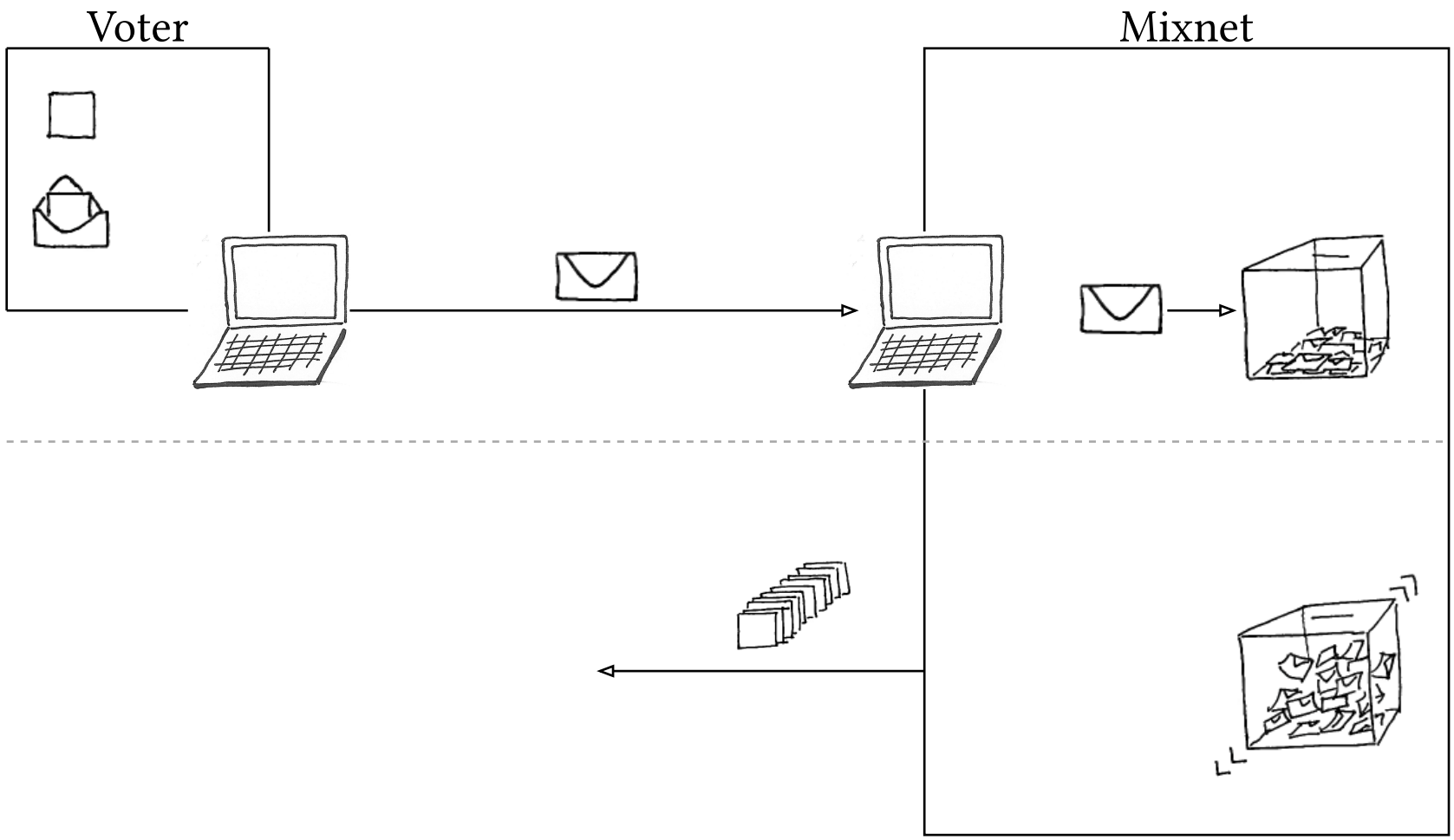

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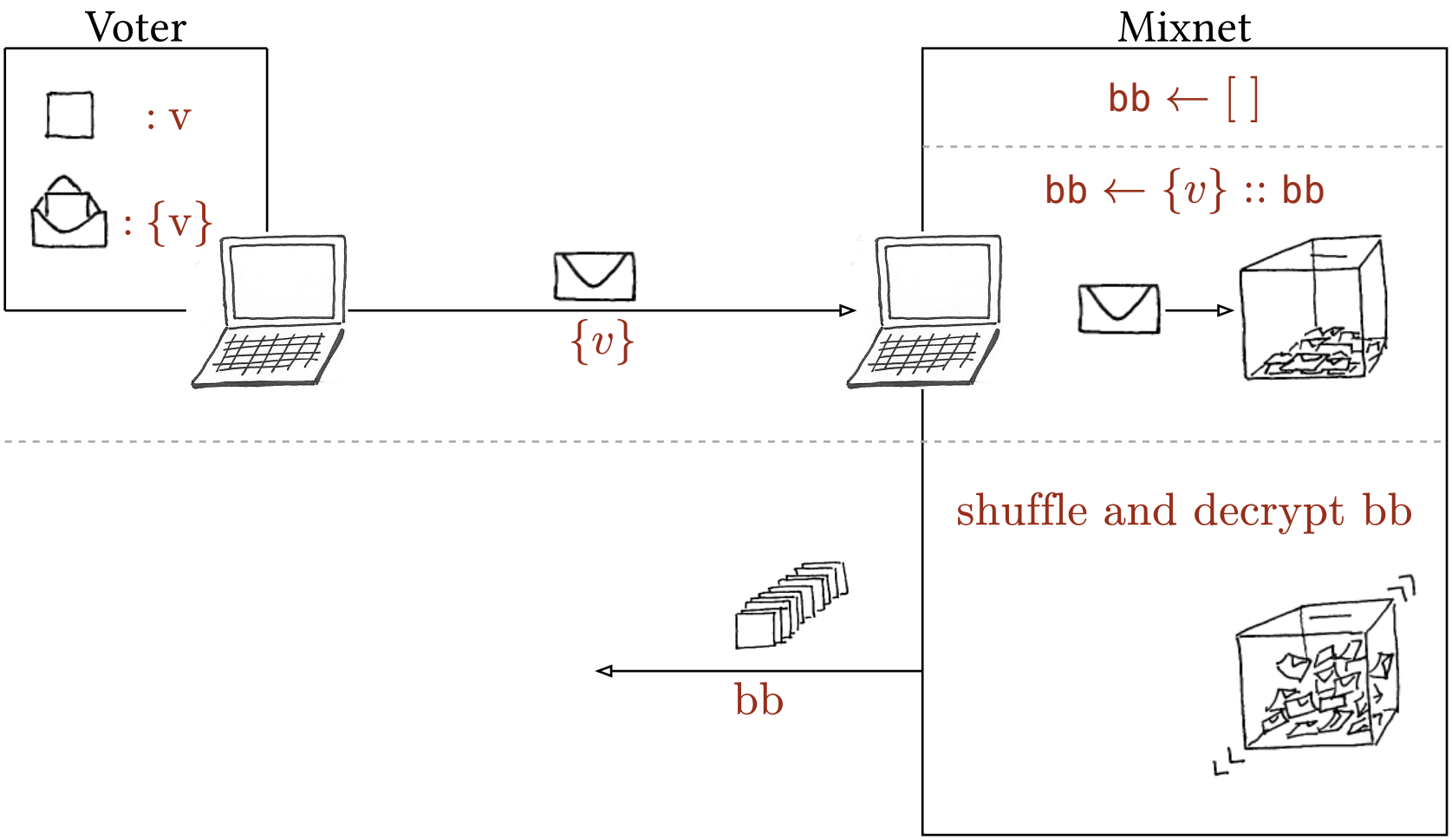

Running example: attacker model and privacy modeling

Privacy

No link possible between voters and votes.

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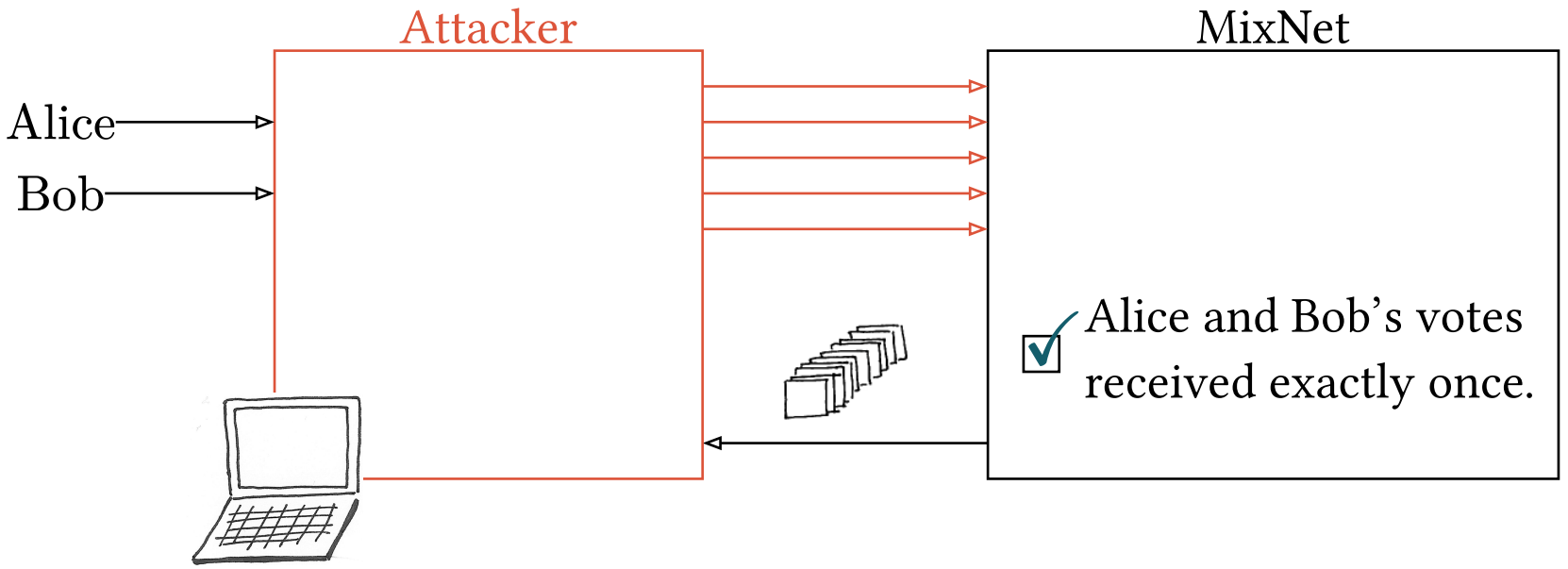

Alice: v_0

Bob: v_1

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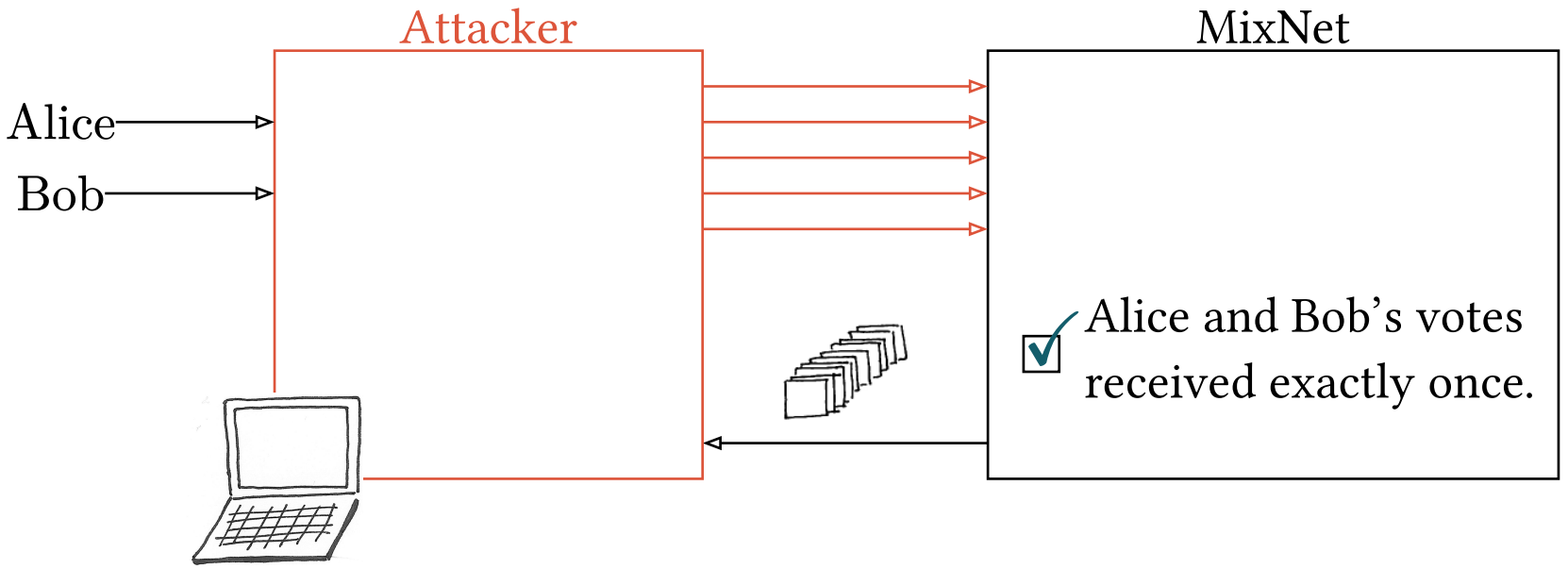

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$\sim$

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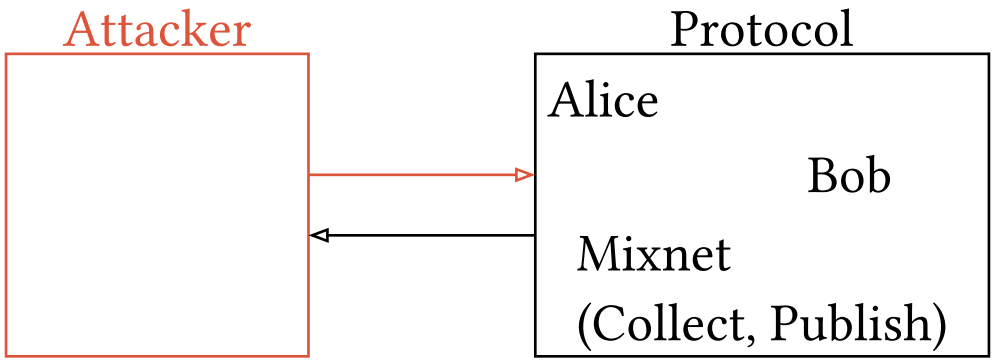
Running example: proof



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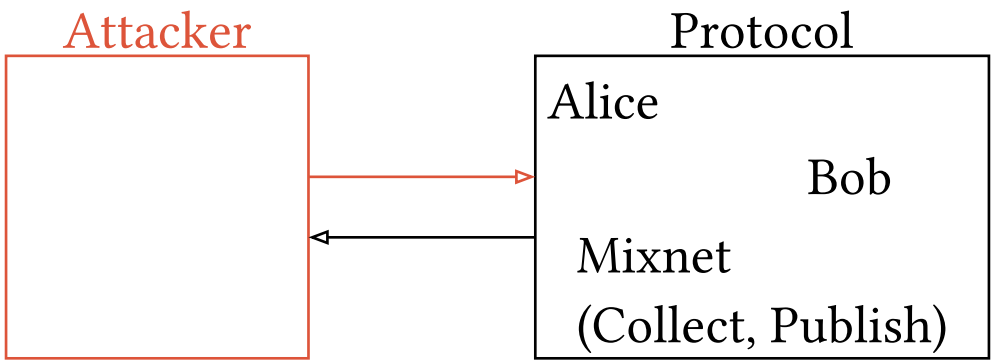

Protocol equivalence: indistinguishability from the attacker's POV

$A : \{ v_0 \}$
$B : \{ v_1 \}$
$P : \text{shuffle}([\ldots, v_0, \ldots, \text{dec}\, v\, \mathsf{sk}, \ldots, v_1, \ldots])$

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Cryptographic proofs

Goal: Formal proof of security

1. **Formal model**: protocol, attackers,...
2. **Cryptographic arguments**: specific proof techniques

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Thesis' framework

- Higher-order CCSA logic (Computationally Complete Symbolic Attacker)
  - Terms: computations (messages (ex: $\{v_0\}$, ...))
  - Formula $t_0 \sim t_1$: computational indistinguishability
  - Inference rules: cryptographic arguments

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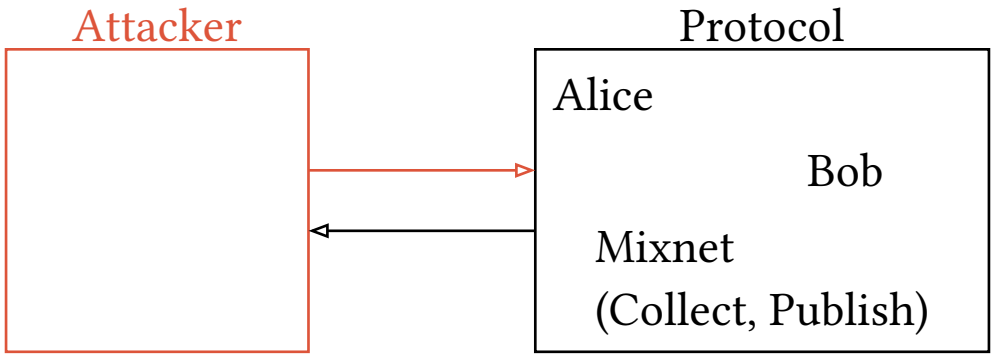
Computer-aided cryptography

- Mechanized proofs: high confidence
- For CCSA: the Squirrel proof assistant

Running example: proof



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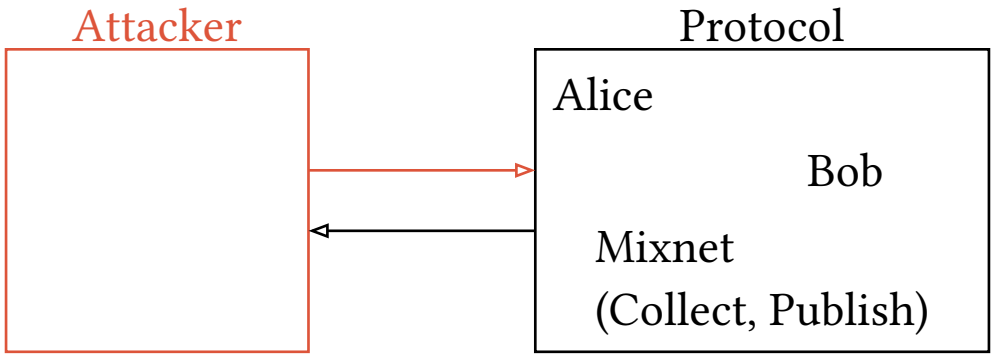

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1: $\{m\}$ hides m

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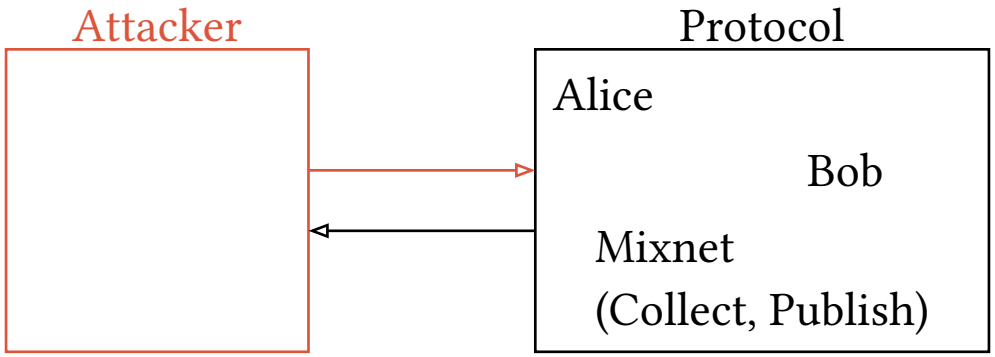

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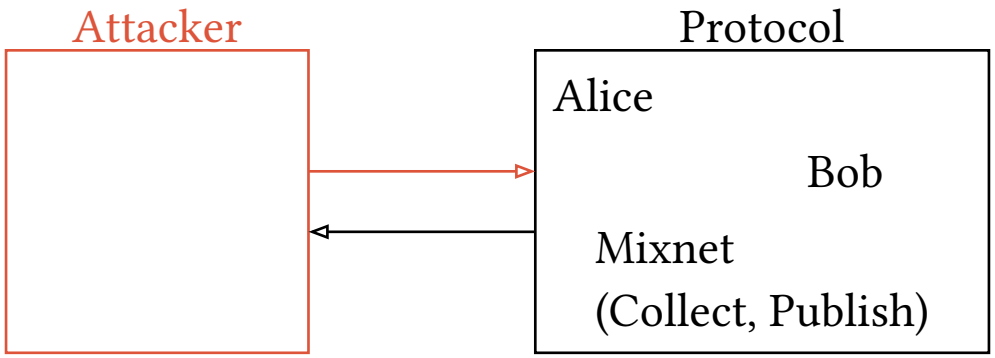

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3: shuffle property

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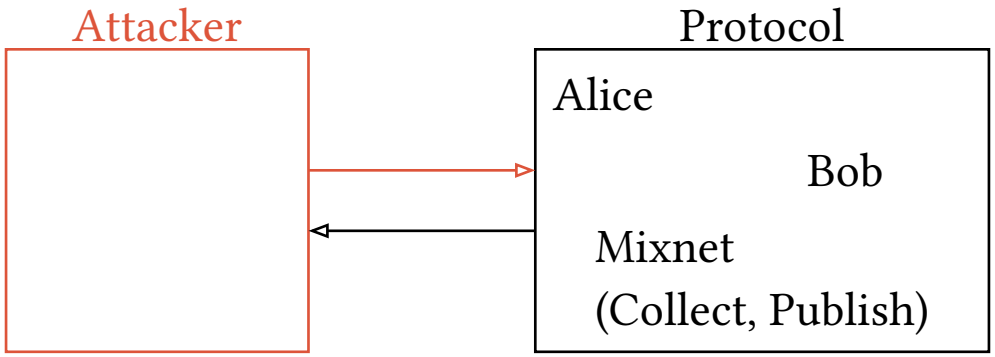

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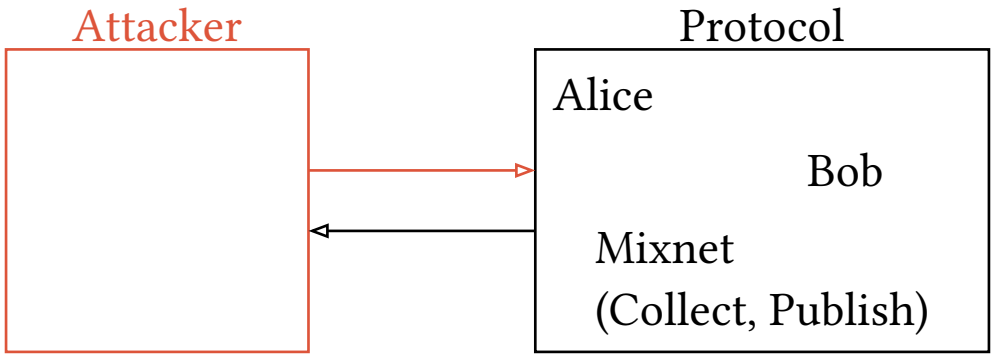

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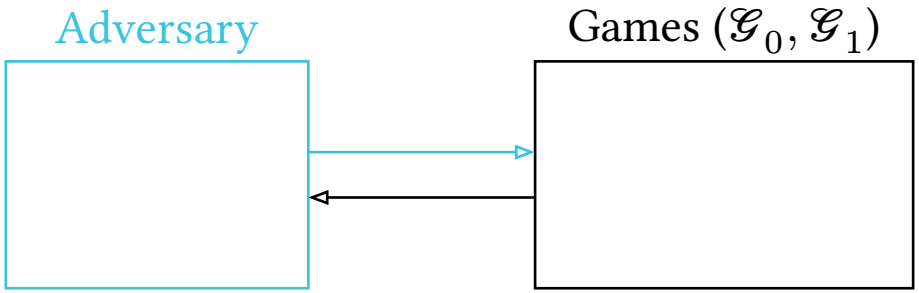
Cryptographic reduction

- from a cryptographic assumption: $\{m\}$ hides m
- to a protocol equivalence.

IND-CCA2 games

Assumption: the encryption "hides" its content.



[1]For fixed-length plain text

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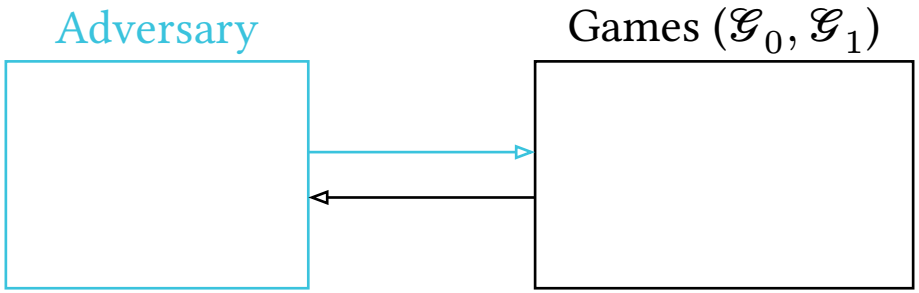

Security parameter: size of the key η

IND-CCA2 assumption

For all **polynomial-time** adversaries $\mathcal{A}$:

$$|\Pr(\mathcal{A}^{\mathcal{G}_0} = 1) - \Pr(\mathcal{A}^{\mathcal{G}_1} = 1)| < \text{negl}(\eta)$$

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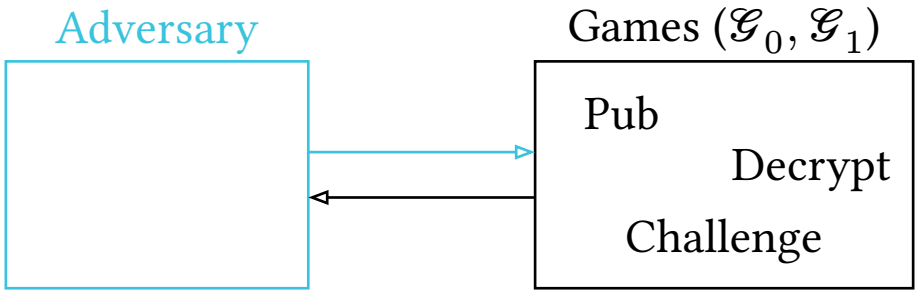

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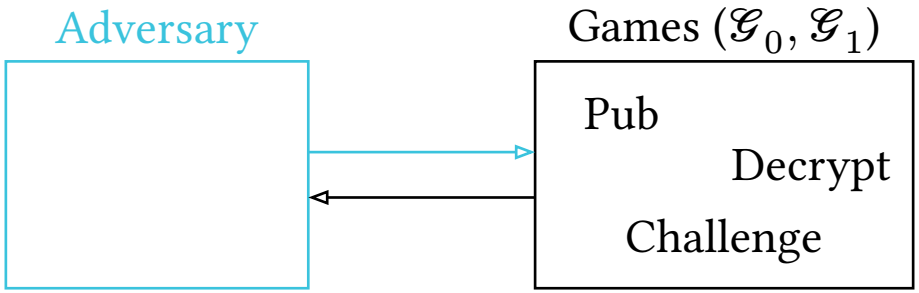

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IND-CCA2 games[1]

$\mathcal{G}_0$: key $\xleftarrow{\$}$

Challenge(m_0, m_1) :=
r $\xleftarrow{\$}$

return $\{ \quad m_0 \quad \}^{\text{r}}_{\text{pk(key)}}$

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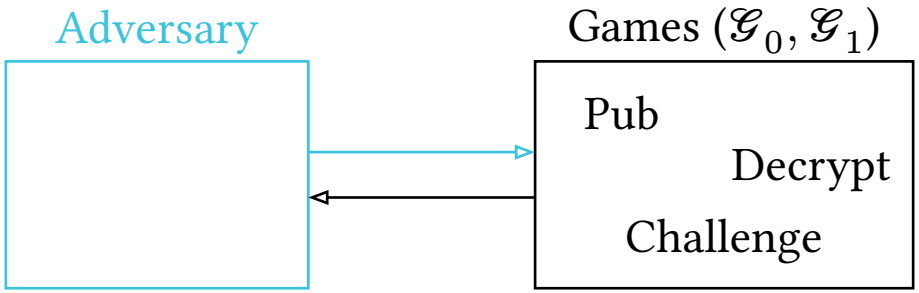

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$\mathcal{G}_1$: key $\xleftarrow{\$}$

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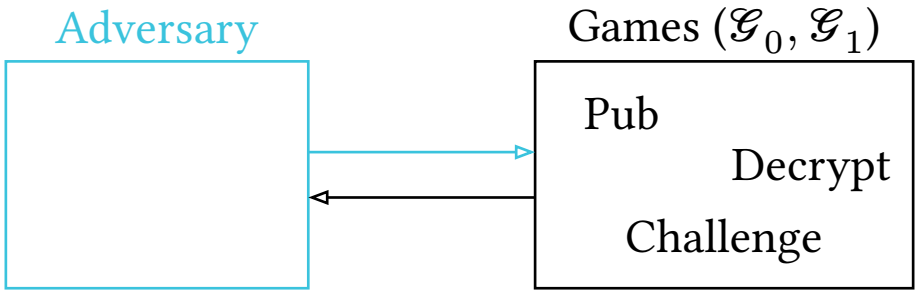

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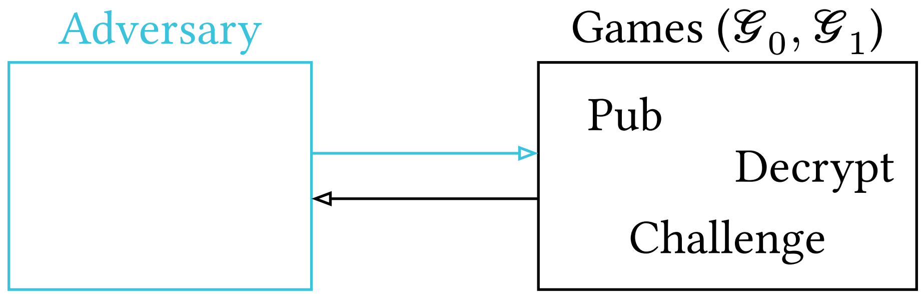

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Pub$() := \mathrm{pk}(\mathrm{key})$

Decrypt$(v) =$ $\mathsf{dec}\, v\, \mathrm{key}$

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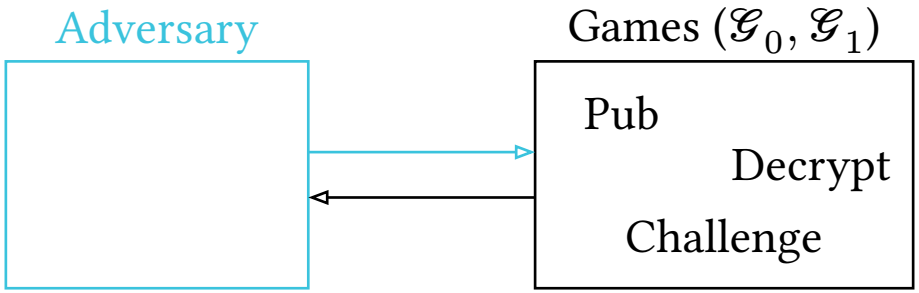

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$\#(\mathcal{G}_0, \mathcal{G}_1)$:

$\text{key} \xleftarrow{\$}$

$\log \leftarrow []$

$\textbf{Pub}() := \text{pk}(\text{key})$

$\textbf{Decrypt}(v) = \texttt{if}\ v \notin \log\ \texttt{then}\ \text{dec}\ v\ \text{key}$

$\textbf{Challenge}(m_0, m_1) :=$

$\text{r} \xleftarrow{\$}$

$\log \leftarrow \log \cup \{\#(m_0, m_1)\}^{\text{r}}_{\text{pk(key)}}$

return $\{\#(m_0, m_1)\}^{\text{r}}_{\text{pk(key)}}$

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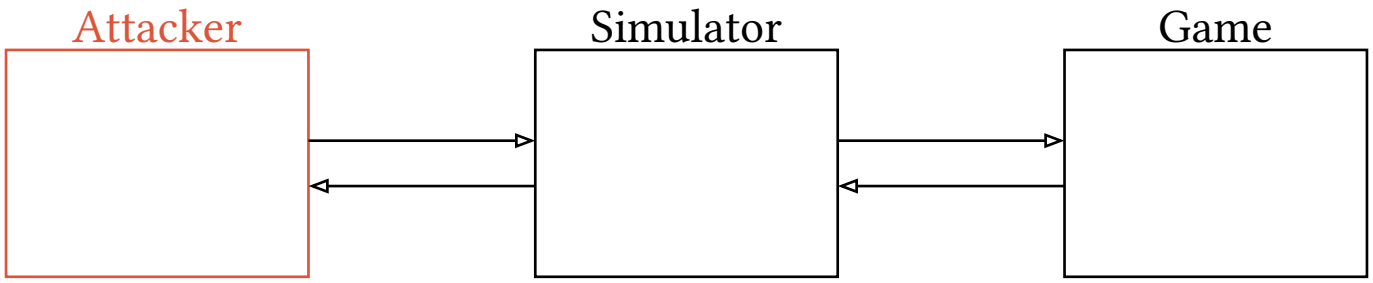
Cryptographic reduction

Cryptographic reduction

Step 1: Build a simulator



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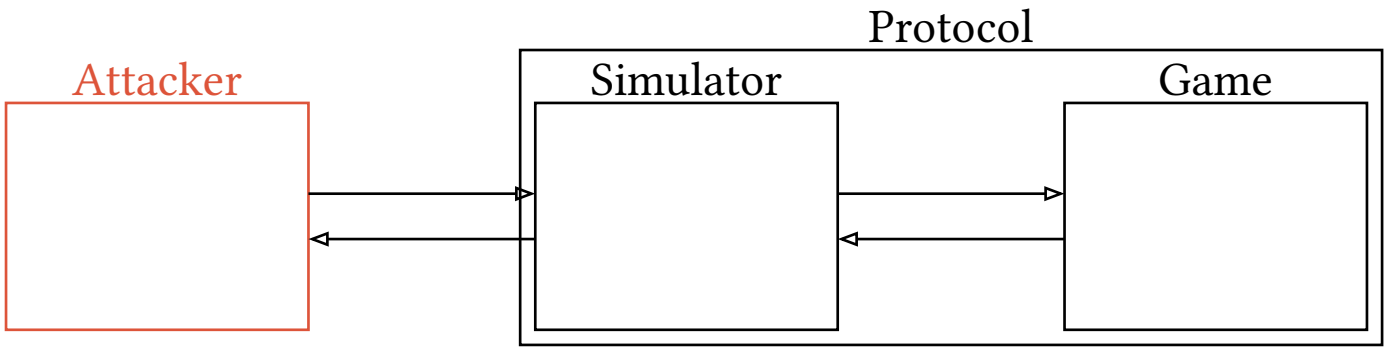

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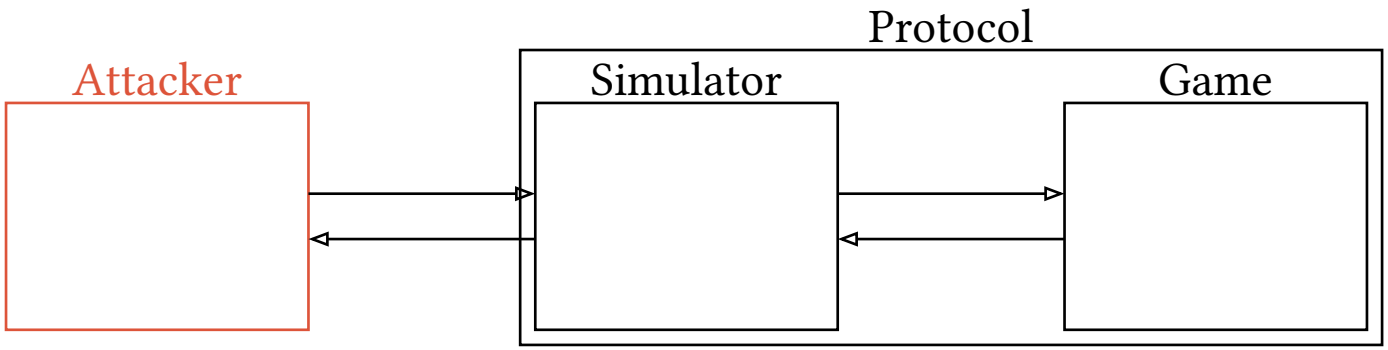

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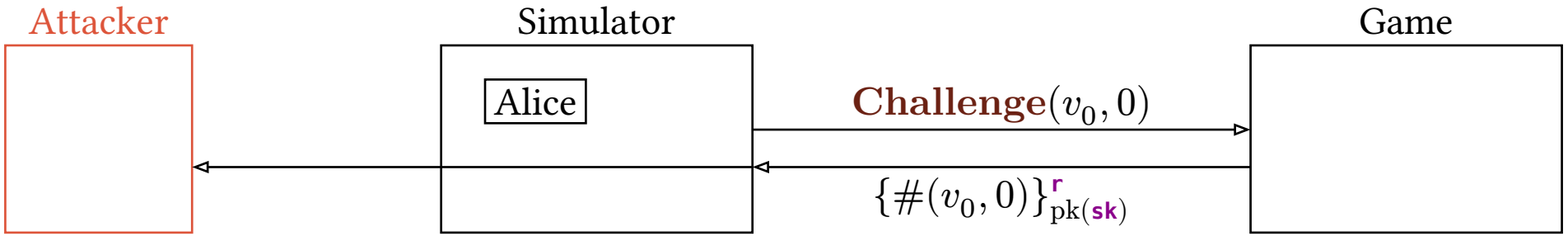

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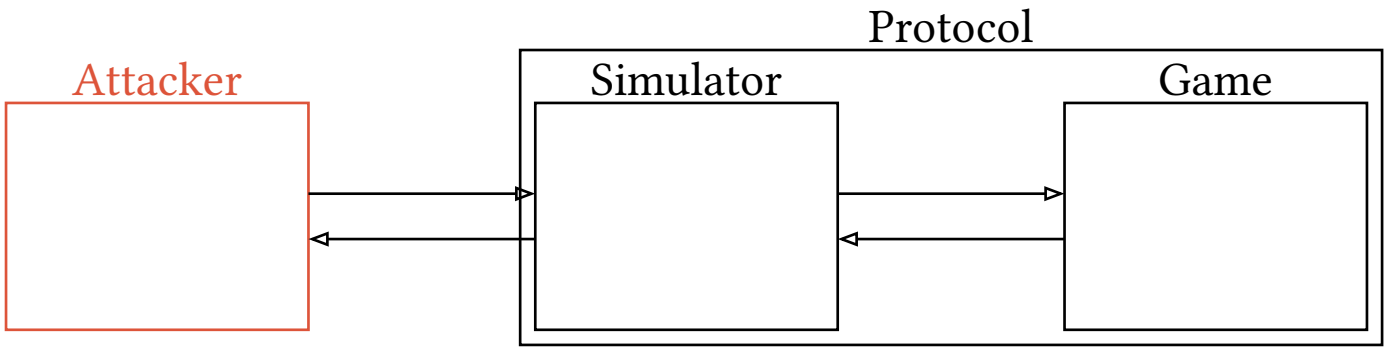

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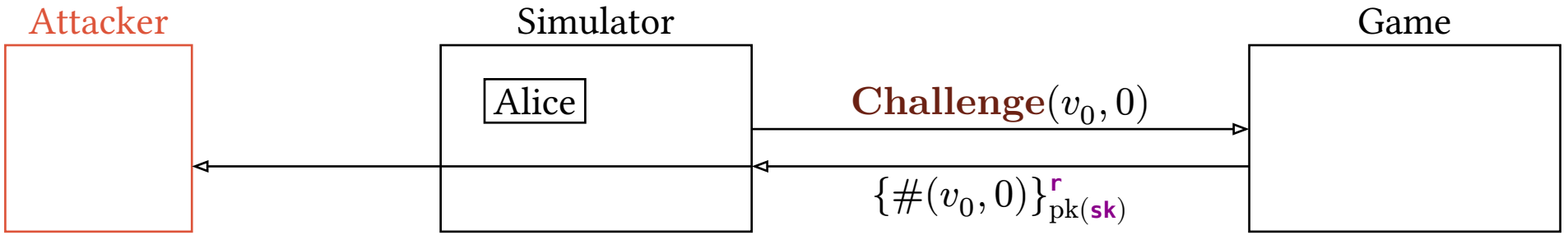
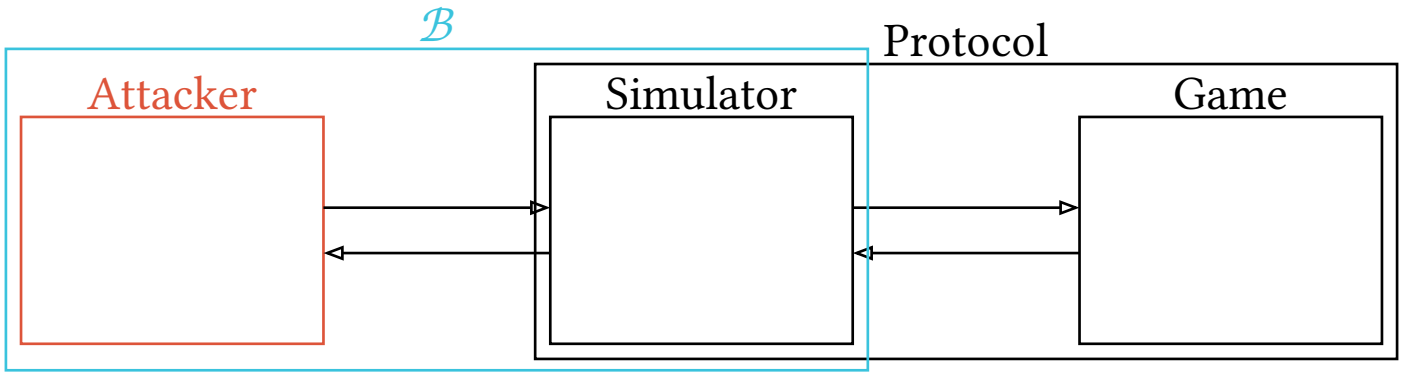


Step 2: Proof by contradiction

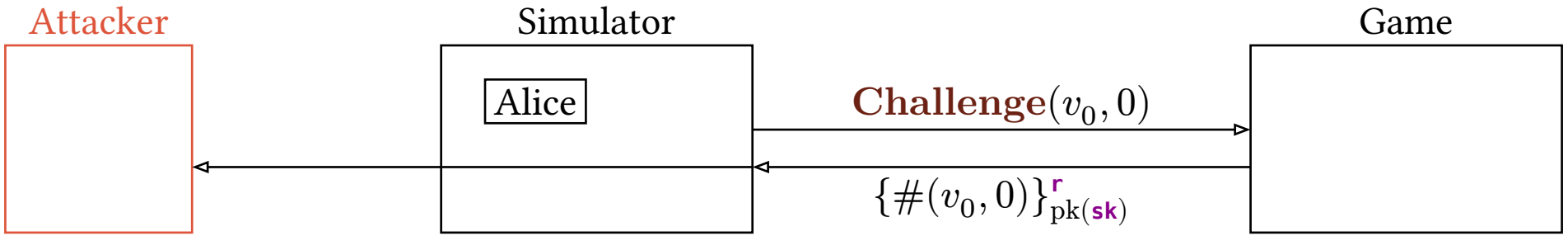
Attacker breaches security of protocol

Cryptographic reduction

Step 1: Build a simulator



For example:

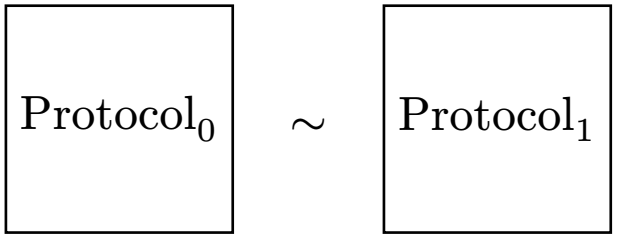


Step 2: Proof by contradiction

Attacker breaches security of protocol $\Rightarrow$ $\mathcal{B}$ breaches security of IND-CCA2.

Simulator synthesis

In summary:



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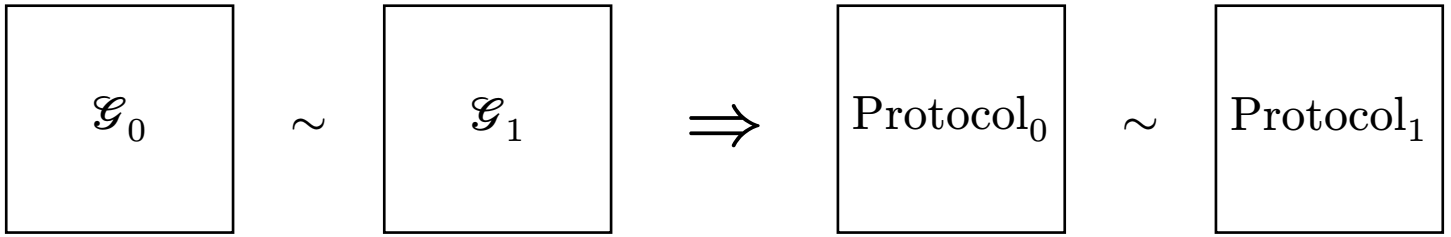

Establishing a cryptographic reduction $\rightarrow$ solving a **simulator synthesis** problem

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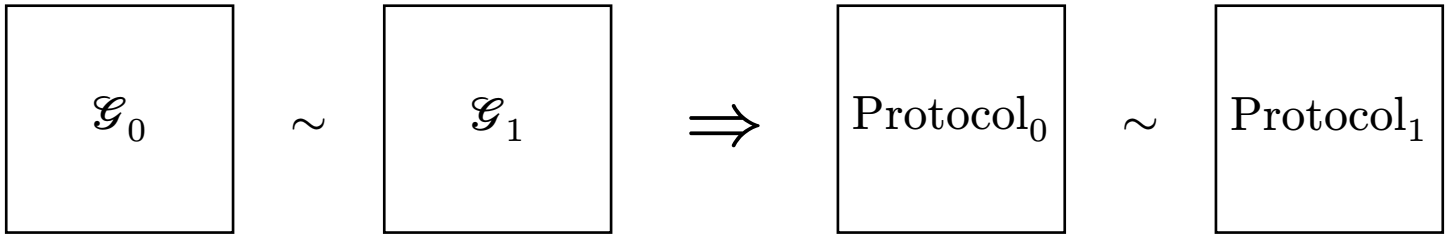

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There exists $\mathcal{S}$ such that $\mathcal{S} \circ \mathcal{G} = \text{Protocol}$

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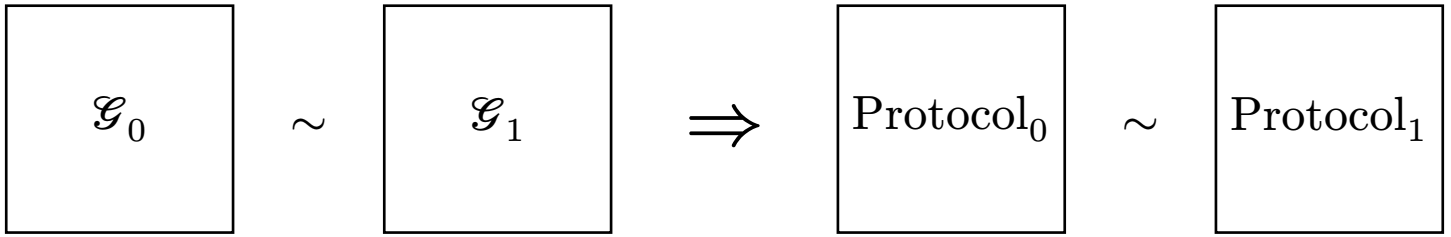

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Reductions in CCSA and Squirrel

Logical rules

- Capture cryptographic assumptions as rules
- Use syntactic checks that ensure the existence of a simulator.

Example:

$$\frac{}{m, \{v\}^{\mathsf{r}}_{\mathrm{pk}(\mathsf{sk})} \sim m, \{0\}^{\mathsf{r}}_{\mathrm{pk}(\mathsf{sk})}} \text{“}\mathsf{sk}\text{ correctly used as key and }\mathsf{r}\text{ fresh in }m\text{ an }v\text{”}$$

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Squirrel

Implement crypto rules as domain-specific tactic (CPA, EUF-CMA, etc)

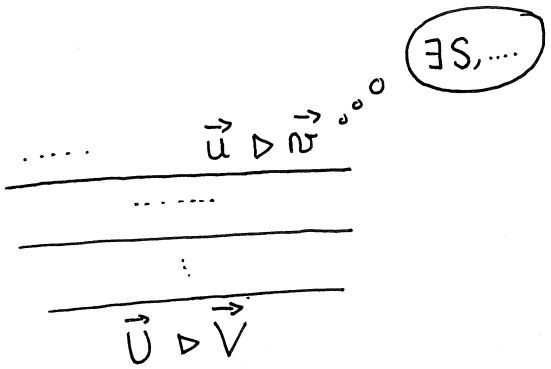
Limit generality and extensibility of Squirrel:

- manually design and prove each new rule
- implement each new rule in **Squirrel**

$\Longrightarrow$ out of reach for standard users and error-prone.

Contributions

- Logical framework for **simulator synthesis**
  - *For arbitrary games*
  - Language for adversaries, games, etc.
  - Bideduction predicate
  - Proof system
-
-
  -



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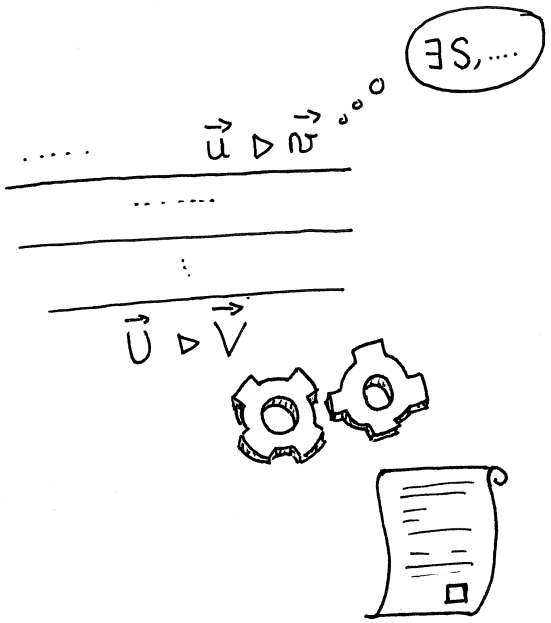

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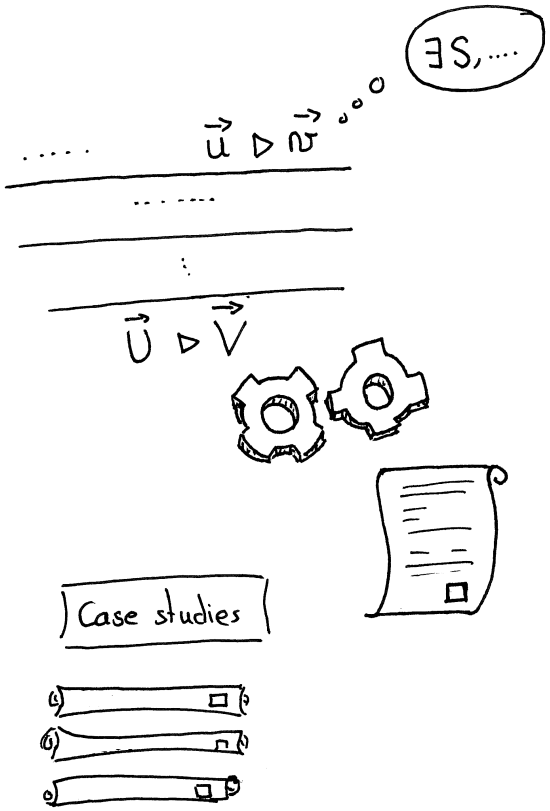

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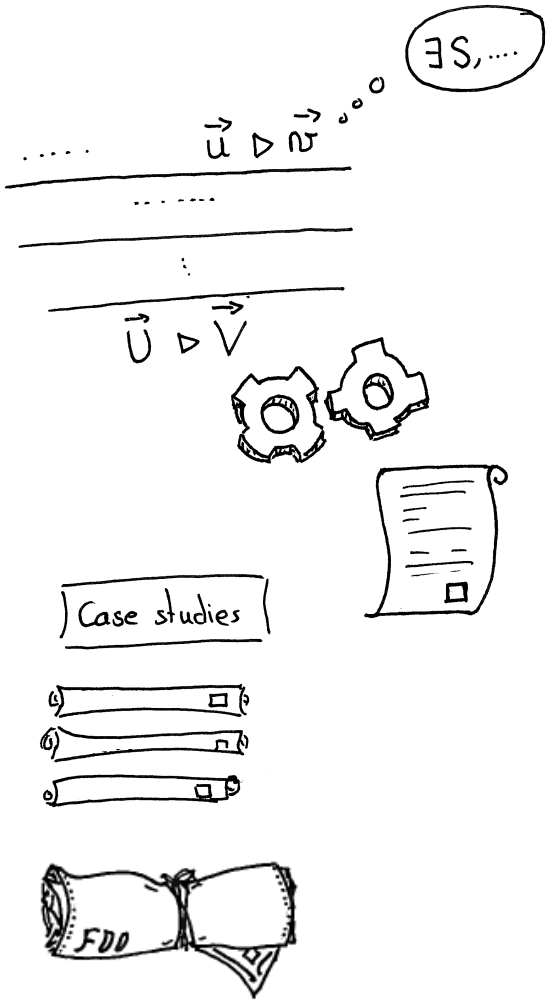
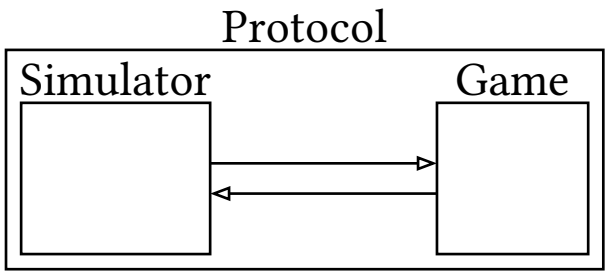
  - Large case study: the proof of vote privacy for FOO e-voting protocol.



3. Bideduction

Starting point

Objectives



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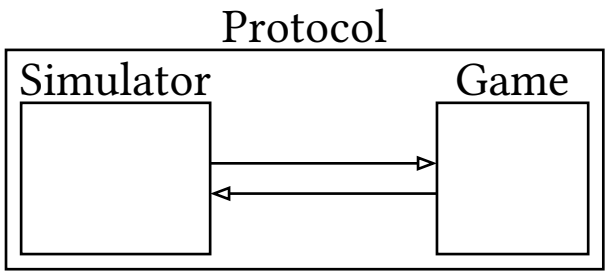

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$$\vdash \#(\vec{u_0}, \vec{u_1}) \rhd \#(\vec{v_0}, \vec{v_1})$$

There exists a (polynomial-time) simulator $\mathcal{S}$ such that: $\mathcal{S}^{\mathcal{G}_0}(\vec{u_0}) = \vec{v_0}$ and $\mathcal{S}^{\mathcal{G}_1}(\vec{u_1}) = \vec{v_1}$.

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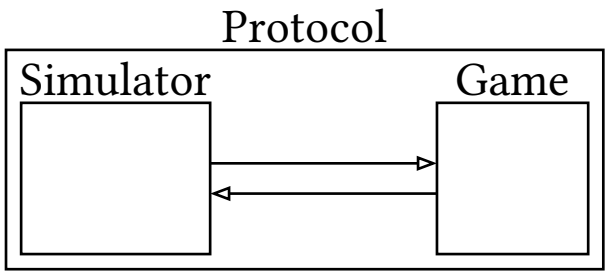

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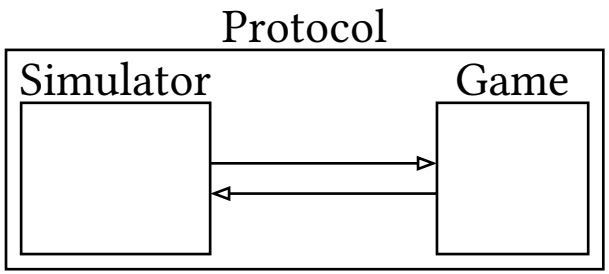

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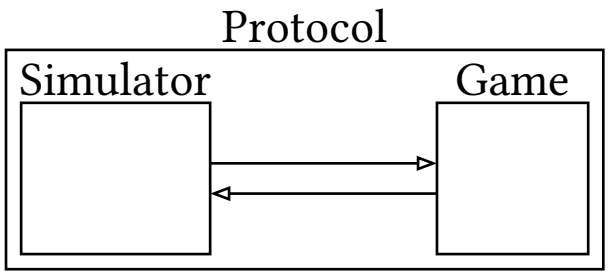
- Rule to capture cryptographic reduction argument (**step 2**)



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Bideduce

$$\frac{\vdash \emptyset \triangleright \#(\vec{v_0}, \vec{v_1})}{v_0 \sim v_1}$$

Intuition: "If there exists such a simulator then v_0 and v_1 are indistinguishable **because $\mathcal{G}_0$ and $\mathcal{G}_1$ are indistinguishable**"

Bideduction rules

What can a simulator do ?

Simulator = probabilistic program with oracle calls.

Bideduction rules

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S(u):= v <- S'(u)
       return f(v)
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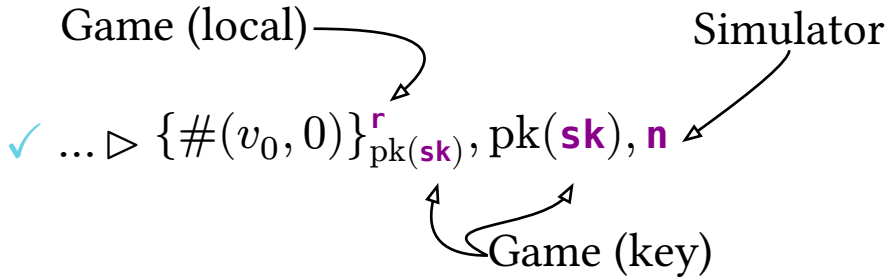

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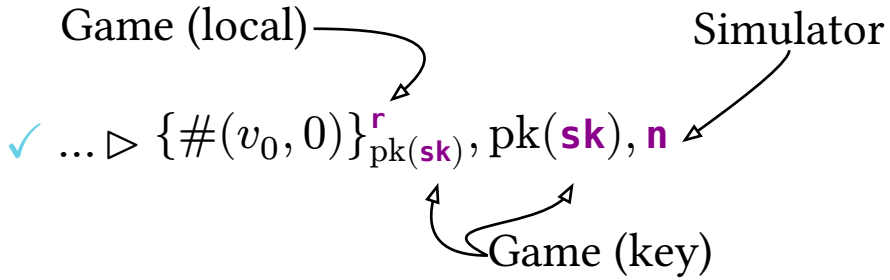

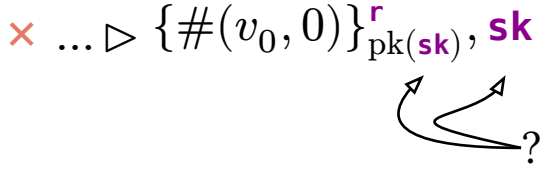

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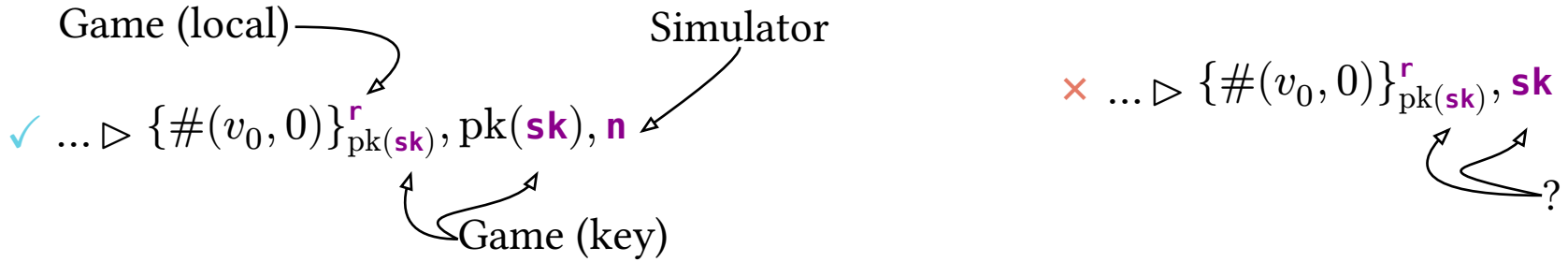

Constraint system

Register randomness ownership.

Tags: S, G.local, G.key,...

Constraint: (n, T)

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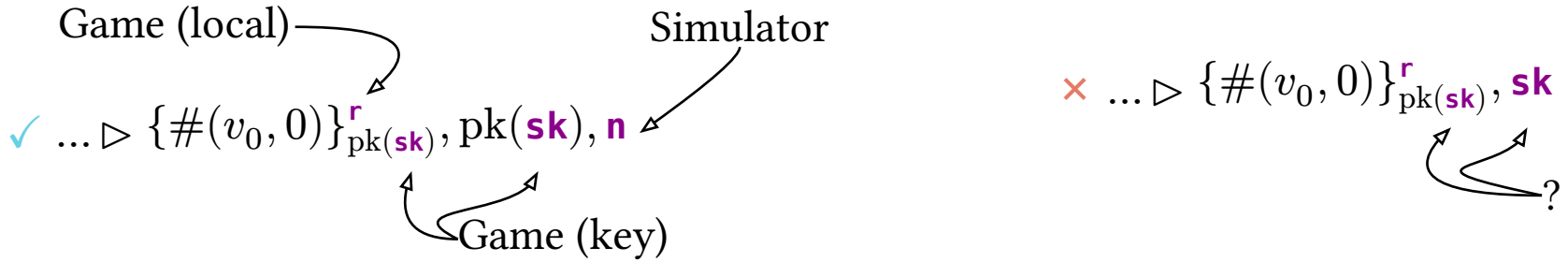

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Random sampling

Sampling rule on example

$$\frac{}{(\mathsf{n}, \mathrm{S}) \vdash \vec{u} \rhd \mathsf{n}}$$

```
S := sample()
```

Bideduction rules

Oracle calls: challenge

Bideduction rules

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$$\textbf{Challenge}(m_0, m_1) :=$$
$$\mathrm{r} \xleftarrow{\$}$$
$$\log += \{\#(m_0, m_1)\}^{\mathrm{r}}_{\mathrm{pk(key)}}$$
$$\text{return } \{\#(m_0, m_1)\}^{\mathrm{r}}_{\mathrm{pk(key)}}$$

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Oracle calls: challenge

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Oracle call on encryption

$$\frac{\mathcal{C} \vdash \vec{u} \rhd v_0, 0}{\mathcal{C}.(\mathtt{r} : \mathrm{G.local}).(\mathtt{sk} : \mathrm{G.key}) \vdash \vec{u} \rhd \{\#(v_0, 0)\}^{\mathtt{r}}_{\mathrm{pk}(\mathtt{sk})}}$$

```
S := v0,z <- S'
     (*oracle call*)
     o <- Challenge v0 z
     return o
```

Semantical challenges

Restrictions on randomness usage:

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$$\frac{\vdash \emptyset \rhd \#(\vec{v_0}, \vec{v_1}) \qquad \mathbf{Valid}(\mathcal{C})}{v_0 \sim v_1}$$

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How do we verify $v \notin \log$?

Solution: Hoare-style pre and post conditions on **game**'s memory.

Tracking memory

$$\mathcal{C}, (\varphi, \psi) \vdash \vec{u} \rhd \vec{v}$$

Pre-condition

Post-condition

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Pre-condition — Post-condition

Example : $\psi := \log \subseteq \{\{\#(v_0, 0)\}^{\mathsf{r}}_{\mathrm{pk}(\mathsf{sk})}\}$

Oracle triples

- Predicate to track game's memory in oracle calls
- Automated verification for sets

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Recursive terms

Squirrel terms: recursive definitions

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Protocol modelling

- Protocol: a sequence of messages (a `frame`)

Recursive terms

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- Mutual definition of
  - `frame`: sequence of messages
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Squirrel terms: recursive definitions

Protocol modelling

- Protocol: a sequence of messages (a `frame`)
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  - `frame`: sequence of messages
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$\texttt{frame}(t) := \langle \texttt{output}(t, \texttt{input}(t)), \texttt{frame}(\texttt{pred}(t)) \rangle$

Proving protocol equivalence: $\forall t, \ldots \vdash \emptyset \rhd \texttt{frame}(t)$

Bideduction: induction

Example:

$$f(x) := t(f(x-1))$$
$$f(0) := t_0$$

If:

- $\mathcal{S}'$ computes $f(x)$ from $f(x-1)$ and x
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Challenges

- Pre and post conditions: **memory invariant $\varphi(x)$**
- Complexity: **iteration over constant-size domain**

Bideduction summary

Bideduction predicate

$$\mathcal{C}, (\varphi, \psi) \vdash \vec{u} \rhd \vec{v}$$

There exists a (polynomial-time) simulator $\mathcal{S}$ such that when $\mathrm{Valid}(\mathcal{C})$:

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Omitted complexity

- Early-sampling semantics: simulator controls full randomness usage.

Bideduction summary

Bideduction predicate

$$\mathcal{C}, (\varphi, \psi) \vdash \vec{u} \rhd \vec{v}$$

There exists a (polynomial-time) simulator $\mathcal{S}$ such that when $\mathrm{Valid}(\mathcal{C})$:

- $\mathcal{S}$ computes $\vec{v}$ from $\vec{u}$
- the game's memory evolves from φ to ψ
- $\mathcal{S}$ controls randomness correctly according to $\mathcal{C}$

Omitted complexity

- Early-sampling semantics: simulator controls full randomness usage.

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- the game's memory evolves from φ to ψ
- $\mathcal{S}$ controls randomness correctly according to $\mathcal{C}$
- $\mathcal{C}$ is well-formned

Omitted complexity

- Early-sampling semantics: simulator controls full randomness usage.
- Lifting simulator computations to probabilities
  - probabilistic couplings
  - well-formedness condition on $\mathcal{C}$

4. Automation

Proof mechanization

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Objective: Automate proof search

$$\ldots \vdash \emptyset \triangleright \texttt{frame}(t)$$

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$$\frac{\ldots \vdash \mathtt{frame}\,\mathtt{pred}(t) \rhd \mathtt{frame}(t)}{\ldots \vdash \emptyset \rhd \mathtt{frame}(t)}\text{Induction}$$

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Objectives

- Heuristic to choose proof rules to apply
- Fill the "..."
  - Propagate downward constraint and post conditions
  - Has to be guessed (induction invariants)

Proof search overview

Synthesis algorithm:

$\mathcal{C}.\square, (\varphi, \square) \vdash \vec{u} \triangleright \vec{v}, \square$

Proof search overview

Synthesis algorithm:

$$\mathcal{C}.\ ,\ (\varphi,\ )\vdash \vec{u}\triangleright\vec{v},\ \rightsquigarrow \Theta,\mathcal{C}',\psi,\vec{w}$$

Proof search overview

Synthesis algorithm:

$$\mathcal{C}, \_, (\varphi, \_) \vdash \vec{u} \triangleright \vec{v}, \_ \rightsquigarrow \Theta, \mathcal{C}', \psi, \vec{w}$$

such that

$\mathcal{C}, \mathcal{C}', (\varphi, \psi) \vdash \vec{u} \triangleright \vec{v}, \vec{w}$ holds assuming Θ holds.

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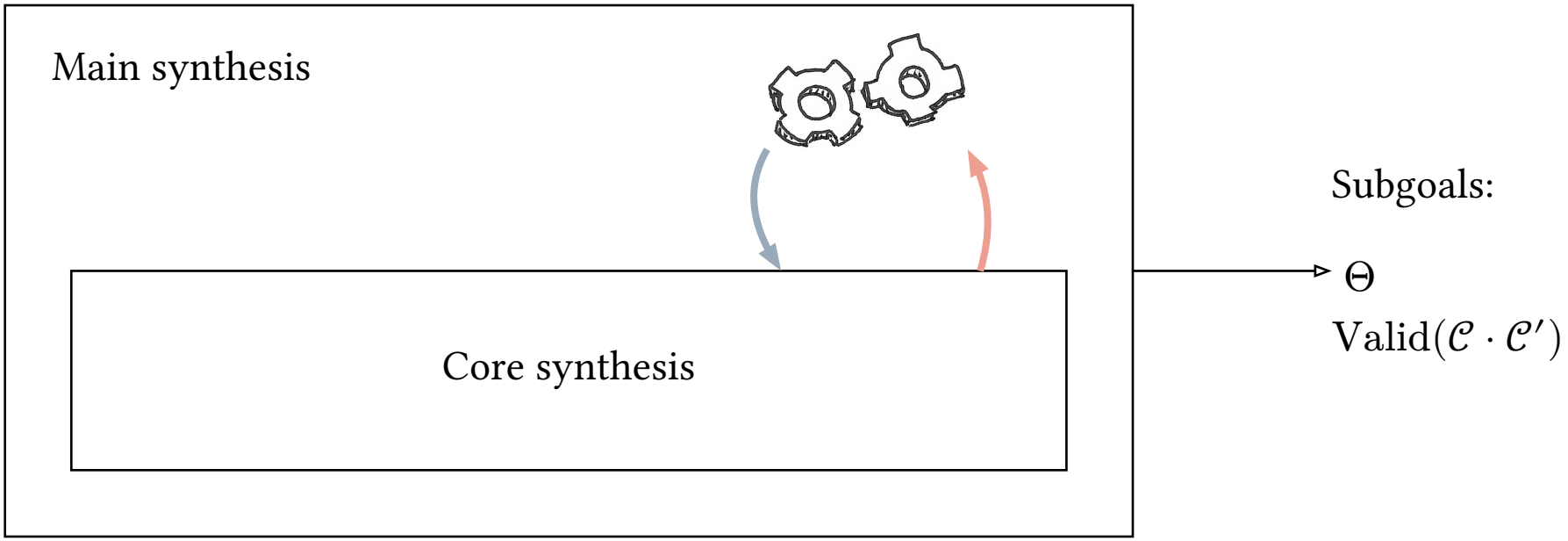


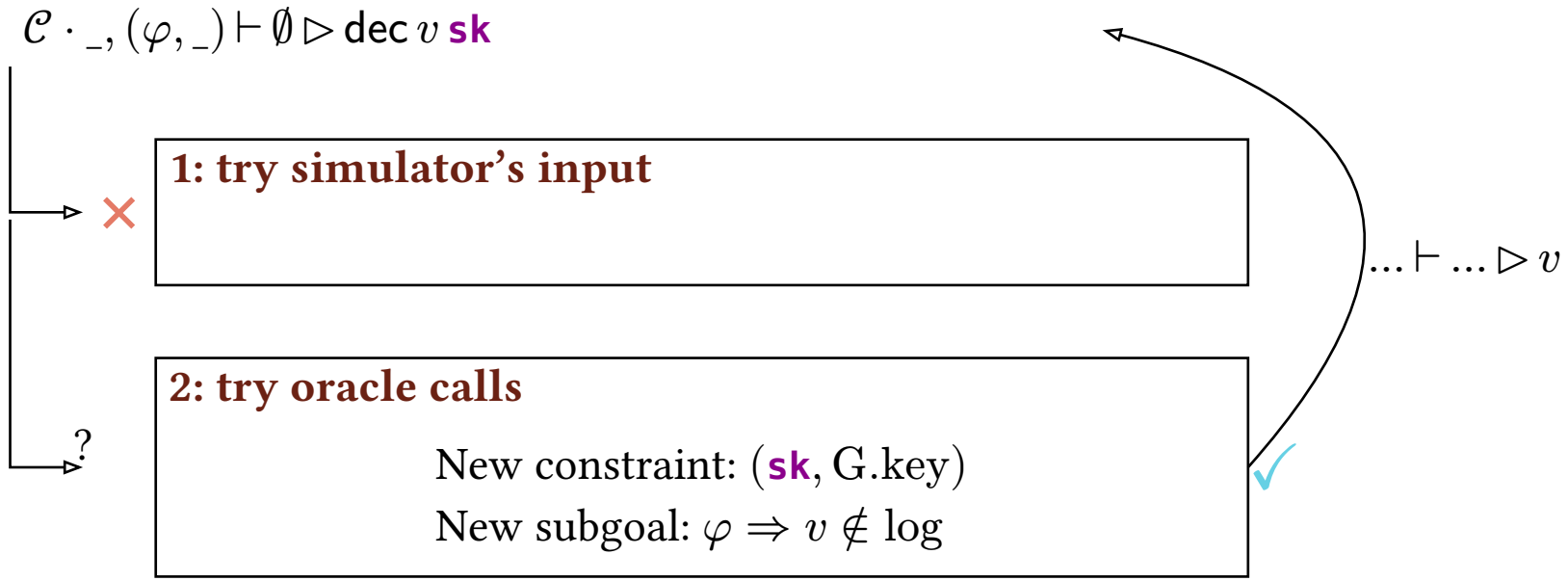
Core synthesis

$$\mathcal{C} \cdot \_, (\varphi, \_) \vdash u \rhd u$$

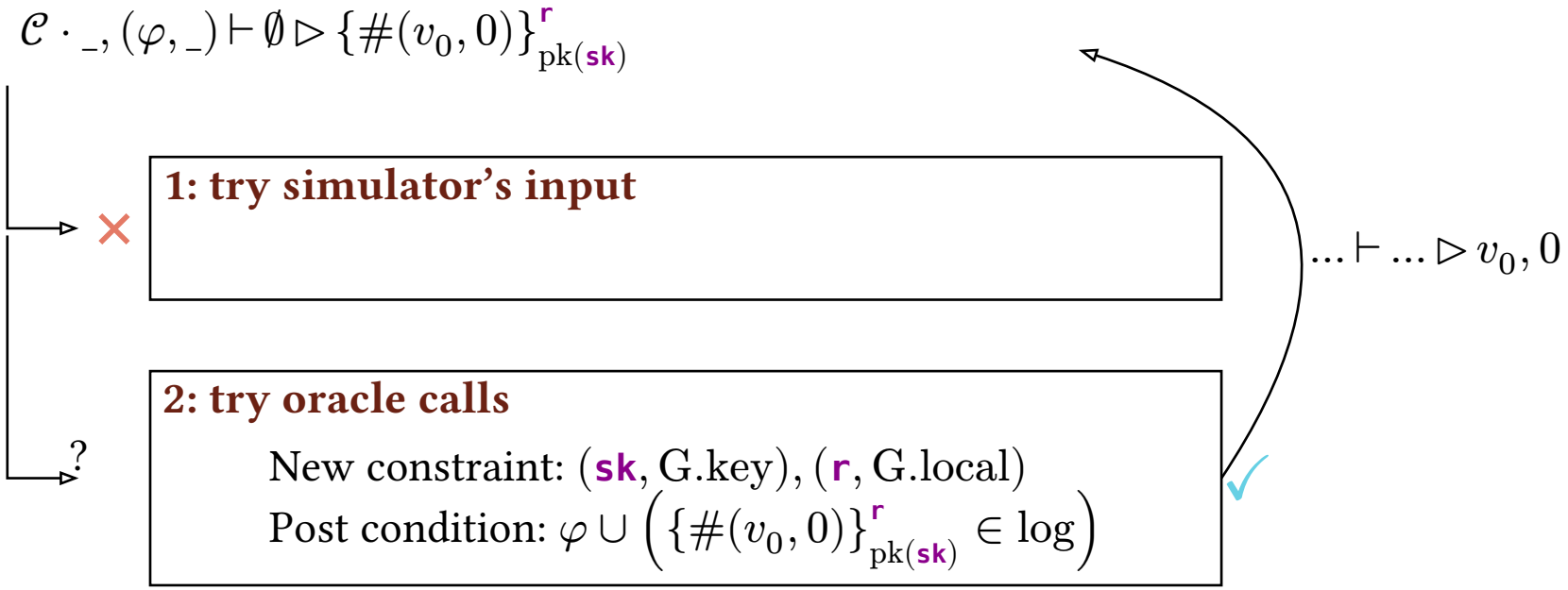
?

1: try simulator's input

Core synthesis



Core synthesis



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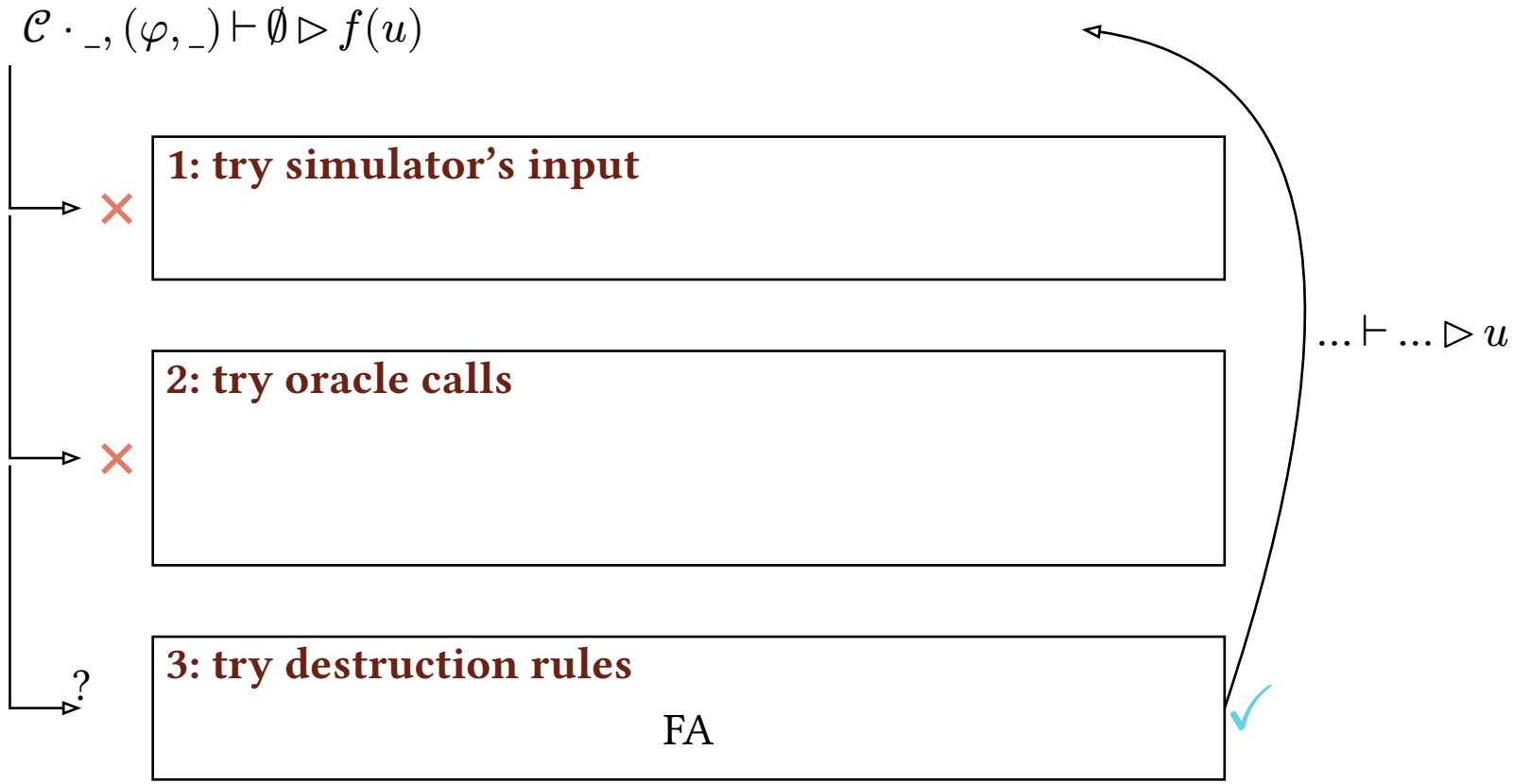
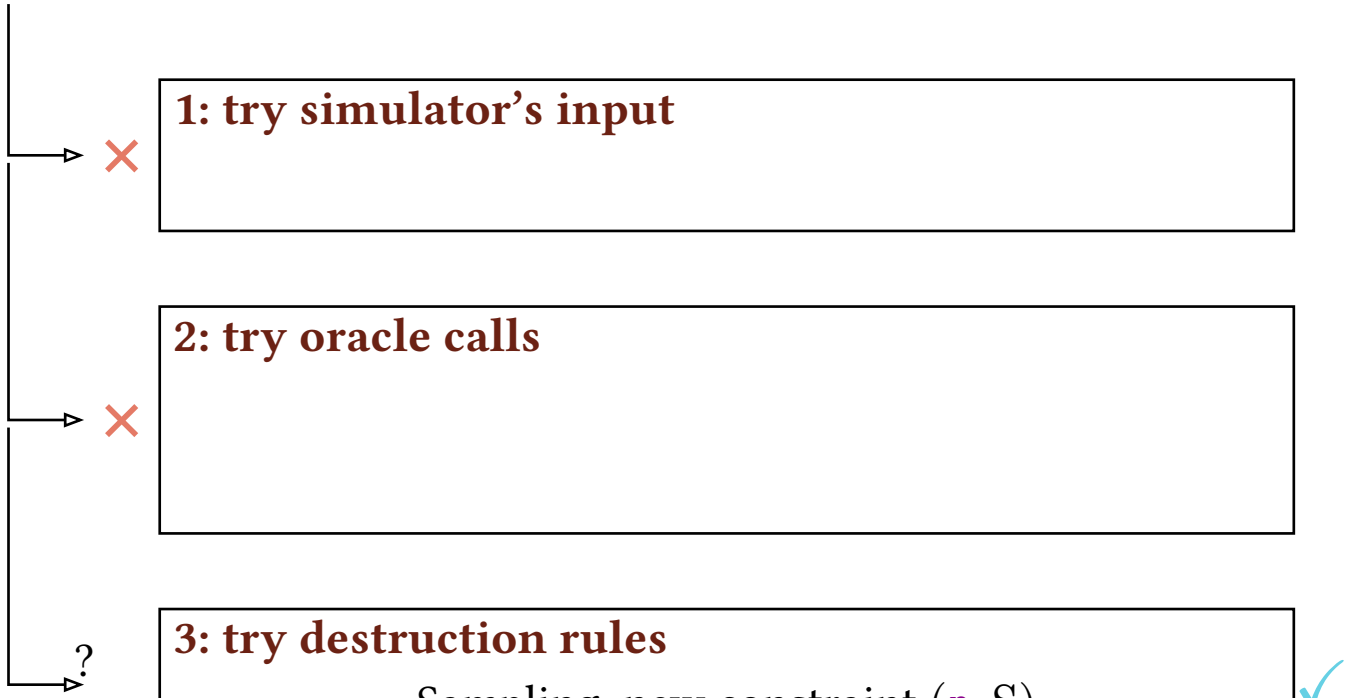


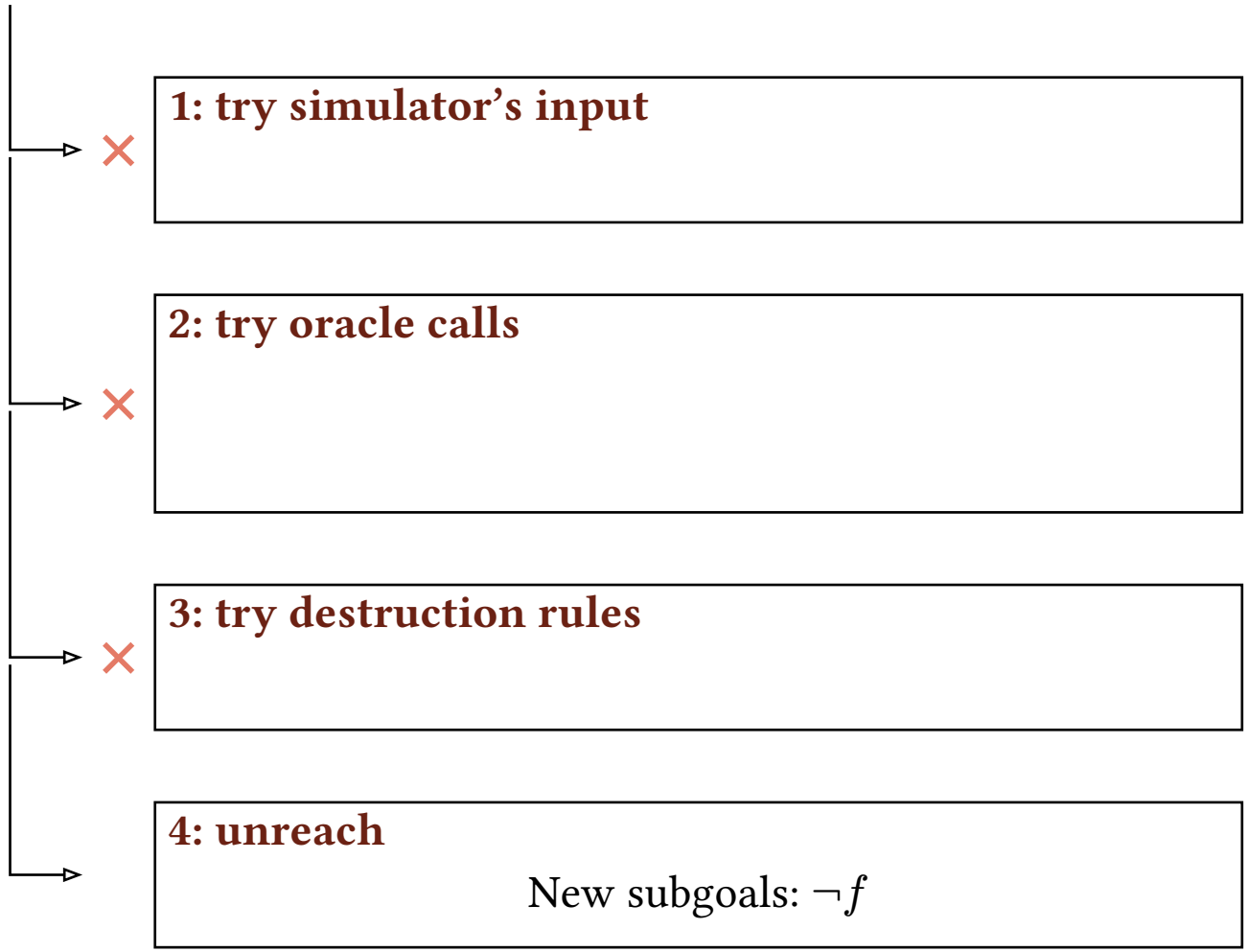
Core synthesis

$\mathcal{C} \cdot \_, (\varphi, \_) \vdash \emptyset \rhd \mathbf{n}$

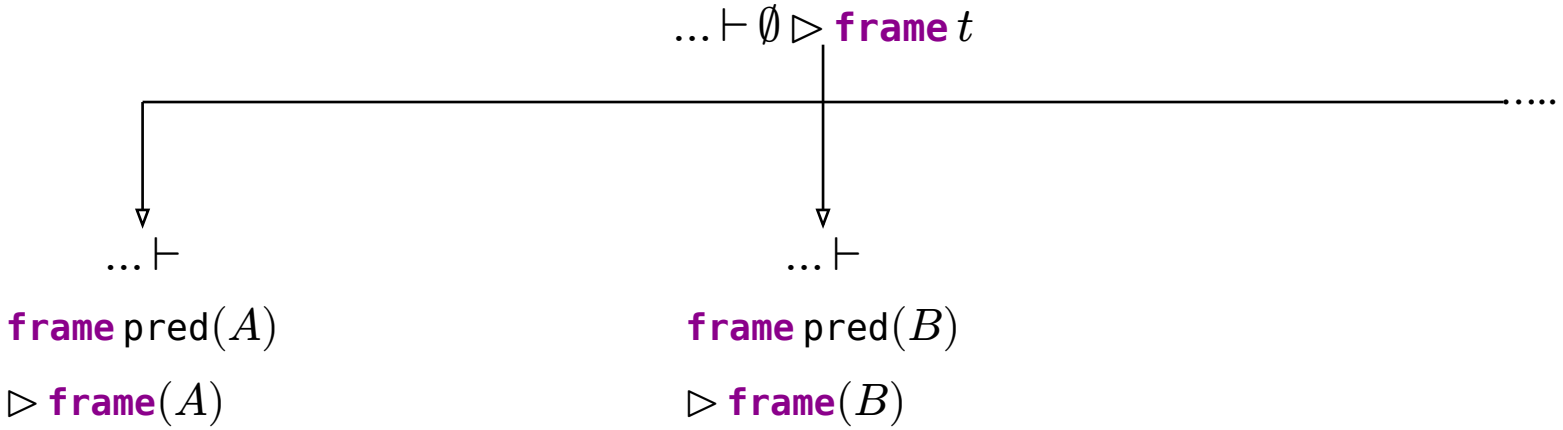


Core synthesis

$\mathcal{C} \cdot \_, (\varphi, \_) \vdash \emptyset \rhd t$ in branching f



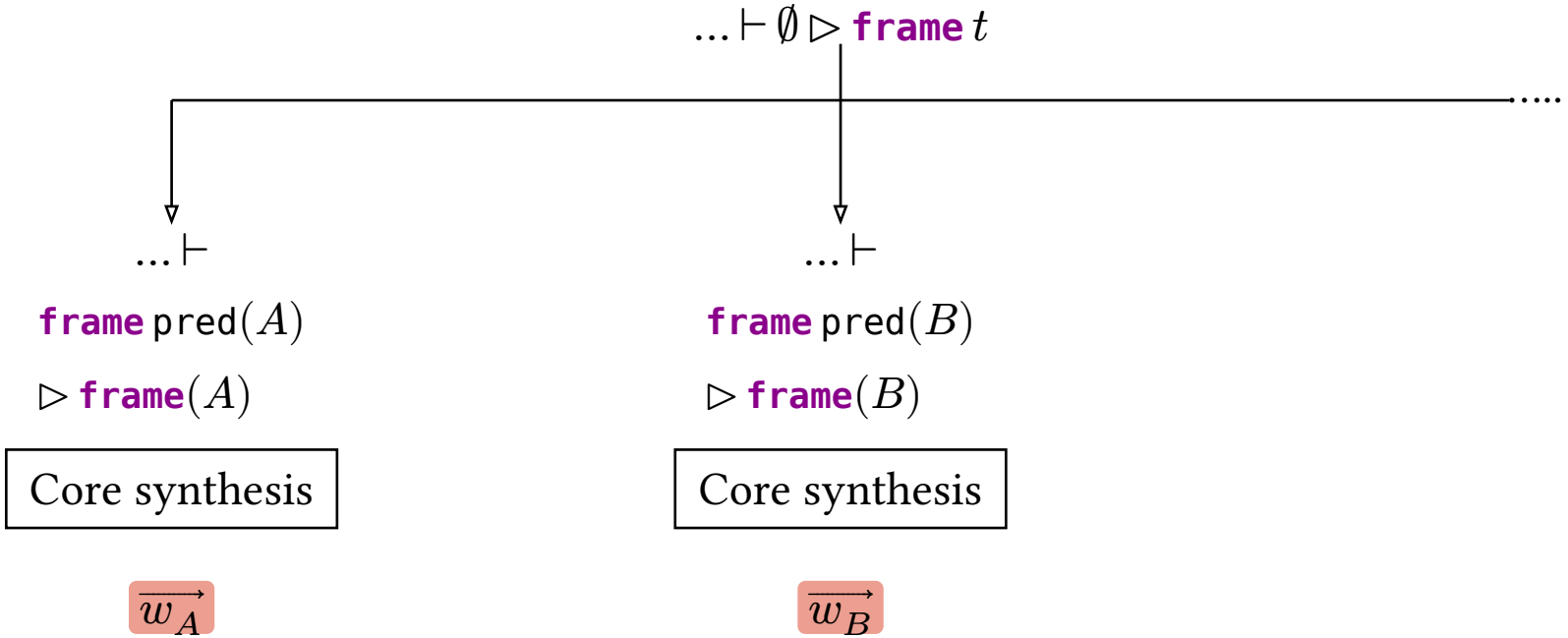
Main synthesis



Init: **Pre-process frame inductions**

- Apply the induction rule for frames
- Case analysis on time points

Main synthesis

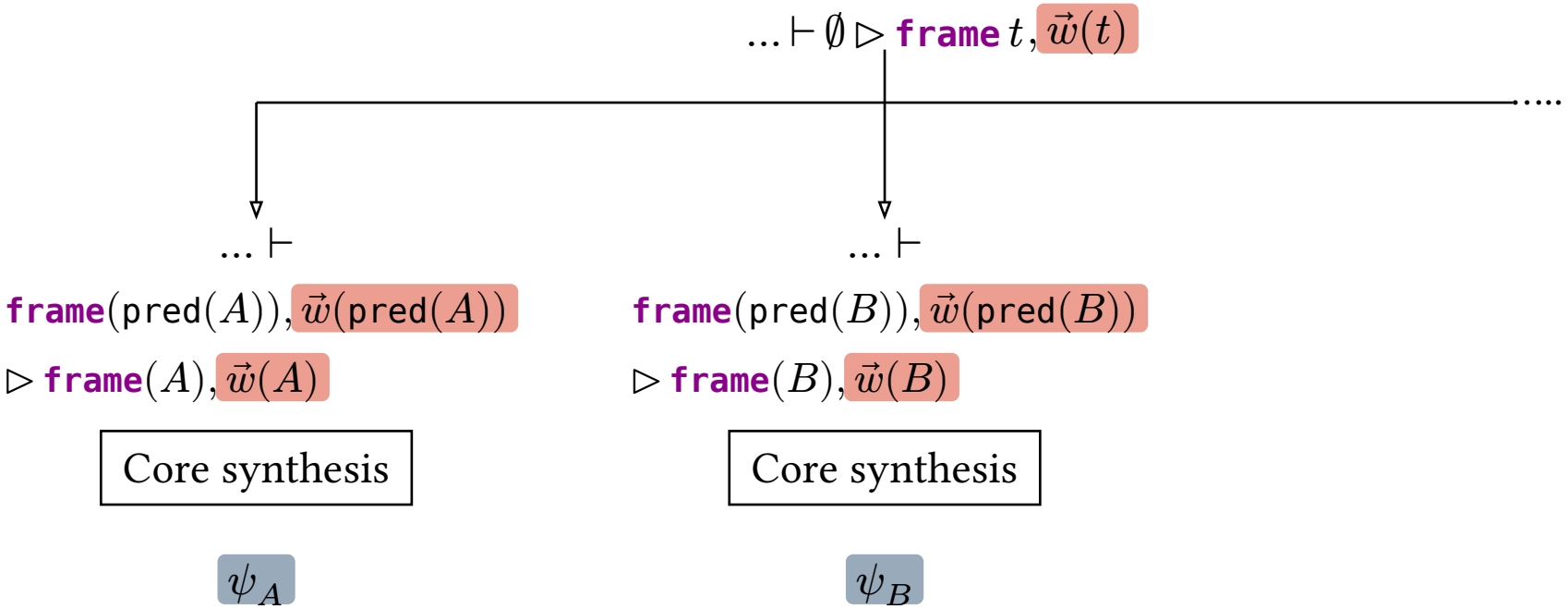


Init: **Pre-process frame inductions**

Run 1: **Memoïzed oracle calls**

- Identifies oracle calls $(\vec{w}_A, \ldots)$
- Terms shared between simulators: $\vec{w}(t) := (\texttt{if}\ A \leq t\ \texttt{then}\ \vec{w}_A, \ldots)$

Main synthesis



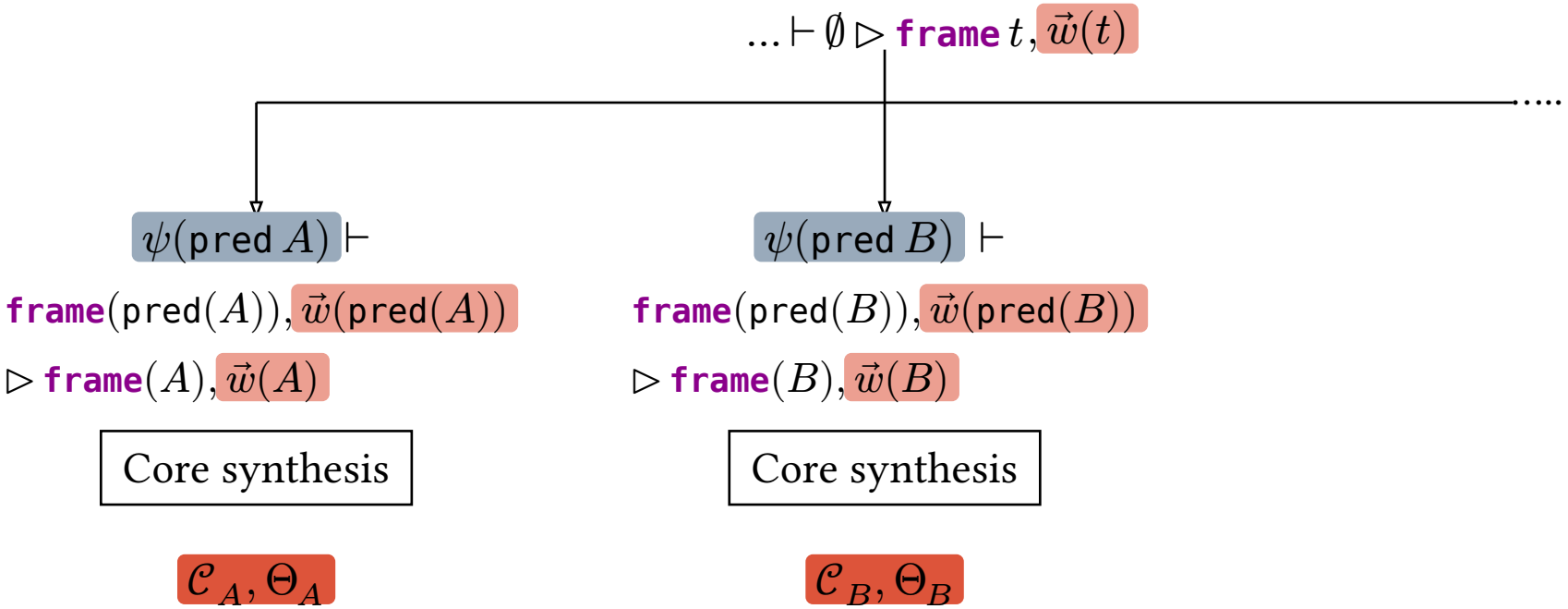
Init: **Pre-process frame inductions**

Run 1: **Memoïzed oracle calls**

Run 2: **Memory invariants**

- Memory invariants $\psi(t) := \texttt{if}\ A \leq t\ \texttt{then}\ \psi_A, \texttt{if}\ \ldots$

Main synthesis



Init: **Pre-process frame inductions**

Run 1: **Memoïzed oracle calls**

Run 2: **Memory invariants**

Run 3: **Constraints and subgoals**

5. Implementation and case studies

Squirrel implementation

What we have:

- Language for games declarations
- Synthesis implementation: `crypto` tactic.

Example:

```
game IND-CCA2 =

  rnd sk;                    oracle challenge m0 m1 = {
  var log = empty_set;         rnd r;
                               var e = enc (#(m0,m1)) r (pk sk);
  oracle pub = {               log:= add e log;
     return pk sk              return e;
  }                          }

                             oracle decrypt c = {
                               return (if not (mem c log) then dec c sk);
                             }
```

Cases studies summary

Protocol	Hypotheses	LoC	Exec time
Hash Lock	PRF	<200	<1s
Basic Hash	EUF-CAM and PRF	<200	<1s
Global CPA	CPA	<200	<1s
Private Authentication	$\text{CPA}_{\$}$	<200	<1s
Partial NSL	IND-CCA2	<200	<1s
NSL	IND-CCA2	$\sim 2.10^3$	$\sim$ 1 min 30

- Replace existing usage of legacy cryptographic tactics
- Execution time kept low in usual cases
- Support new hypotheses

Focus on FOO

Protocol	Hypotheses	LoC	Exec time
FOO	IND-CCA2, Blinding Commitment Hiding	$\sim 10^4$	~ 4 min 20

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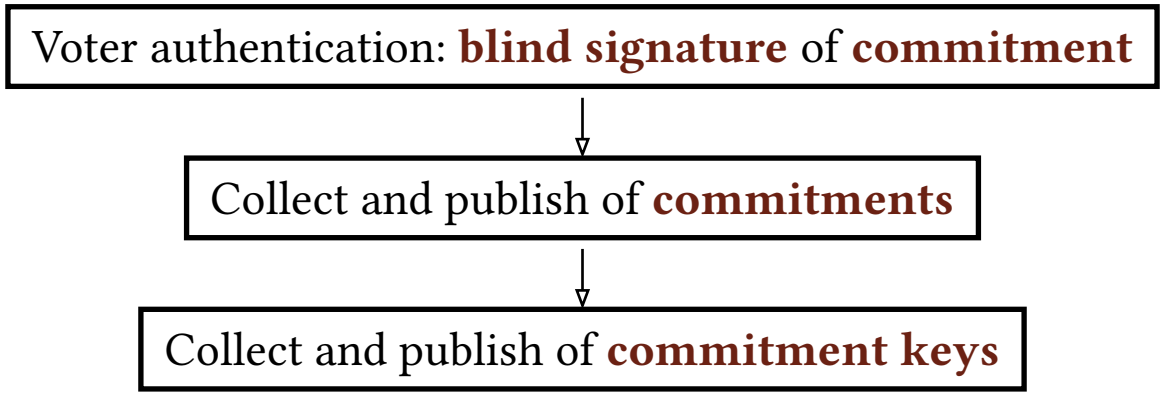

Proof structure:
1. IND-CCA2
2. Shuffle + **deductions** to reduce the protocol
3. Cryptographic reduction on **commitments** and **blind signatures**

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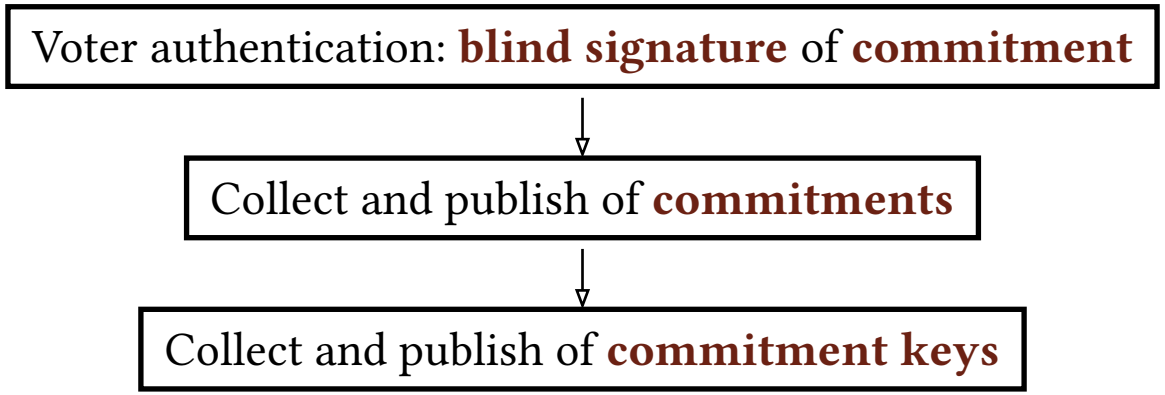

Proof structure:
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Execution time: realistic use of the tactics:

- Most calls are $< 0.5s$
- Maximum: 11s

6. Conclusion

What has been done

- Framework for simulator synthesis
  - Bideduction predicate and proof system
  - Structure to support composability:
    - constraint system
    - pre and post conditions

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- Automation
  - Goal- and constraints-directed
  - Infer memoïzing simulators
- Implementation
  - Integrated in Squirrel's stream
  - Application to simple protocols
    - Replace existing axioms, and support new assumptions
  - Large-scale validation with proof of FOO

Limitations and future work

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- More permissive semantics (approximate couplings)
- Enrich pre and post conditions logics (maps)

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General new directions

- Inferring game hops
- Debate on full automation: strong constraints on running time, limiting improvements
- Transfer to other frameworks (ex: EasyCrypt)